Persistency of Cointegration Relationship for Spot and Futures Prices of WTI and Brent Crudes

Hyun Jin Jang*, Xiao Pan†, Robert I. Webb‡

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Abstract

We examine the persistency of the long-run relationship for the quadruple price dynamics of the West Texas Intermediate (WTI) and Brent crude spots and their futures contract prices. We conduct a cointegration test for the six pairs of the four time-series via full-sample and sub-sample tests on the period from 1993 to 2022, based on the vector error correction method and the structural break test. We also verify price leadership in asset dynamics employing the permanent-transitory (PT) decomposition analysis. The full-sample test shows that the four markets are pairwise cointegrated, while the sub-sample test illustrates that the bivariate cointegration patterns are not only inhomogeneous but also time-varying and non-linear. With the sub-sample results, we evaluate the persistency of a long-run relationship to the six pairs, resulting in that the WTI and Brent spot prices are at the highest level, the combination for the crude and its futures are at the second highest, yet the WTI and Brent futures are at the lowest. The PT analysis shows that for the recent half decade, the WTI spot dominates the Brent spot in the crude market, while Brent futures leads WTI futures in the derivatives market. We also discuss the empirical relationship between cointegration and correlation.

keyword: Cointegration; Vector error correction method; Permanent-transitory decomposition; West Texas Intermediate; Brent; Persistency of long-run relationship

*Corresponding author, School of Business Administration, Ulsan National Institute of Science and Technology (UNIST), Ulsan 44919, Republic of Korea, E-mail: janghj@unist.ac.kr

†School of Business Administration, UNIST, Ulsan 44919, Republic of Korea

‡McIntire School of Commerce, University of Virginia, Charlottesville, VA 22903, USA

1 Introduction

This study investigates the persistency of the cointegration relationship in quadruple asset dynamics, that is, the spot and futures prices of West Texas Intermediate (WTI) and Brent crude oils. It is well known that the four-price series have prolonged persistent long-run relationships, because not only are two crudes closely associated due to their similar chemical grades and roles in energy markets, but the crude spot and its futures cannot stay apart owing to the nature and function of futures contracts. We examine the cointegration relationship among the four time-series based on the vector error correction method (VECM) with a structural break test for the test period from 1993 to 2022. We also verify price leadership in the asset dynamics by employing a permanent-transitory (PT) decomposition analysis.

WTI and Brent are the most widely used benchmarks for global crude oil markets. WTI serves as a benchmark for crude oil produced in the US; however, it is carefully monitored by traders, analysts, and policymakers worldwide. Brent is considered a global benchmark for crude oil pricing and is used to value approximately two-thirds of the world's internationally traded crude oil. Both WTI and Brent crudes are equipped with well-established financial markets and their futures markets are known for having the most liquid commodity contracts in their respective marketplaces.

While both WTI and Brent crudes are used as indicators of global oil prices and are sophisticatedly financialized as electronic markets accessible to anyone, they exhibit differences in pricing due to external factors such as quality variations, transportation costs, regional supply/demand imbalances, and geopolitical risks. WTI and Brent are considered high-quality crudes, which are sweet and light due to high API gravity and low sulfur contained[1], but WTI crude is typically slightly sweeter and lighter than Brent. The WTI delivery point is a landlocked area and is usually far from where the ultimate demand exists, as WTI extracted from land-based oil fields is transported through pipelines to storage terminals in Cushing, Oklahoma. However, Brent crude is delivered to storage terminals located in the Shetland Islands after being produced from under-water oil fields in the North Sea. As Brent is immediately transported to vessels employing a just-in-time production scheme, it is relatively easier to transport it to distant locations. For these reasons, WTI price tends to be more sensitive to infrastructure issues, US domestic supply/demand factors, and relevant policies. However, Brent prices are prone to be influenced by supply/demand dynamics in broader regions, including Europe, Africa, and Asia, and are also more sensitive to geopolitical tensions with other oil-producing countries (Maslyuka and

[1] Low density and sulfur crudes are usually easier to refine into diesel fuel, gasoline, and other high-demand petrochemical products.

Smyth, 2009; Fattouh, 2010; Scheitrum et al., 2018).

Conceptually, the law of one price states that WTI and Brent crude prices can differ by no more than the factors caused by disparate production and storage models because the quality of the crudes is largely homogeneous, as mentioned above. Theoretically, a futures and its underlying spot prices should hold a parity relationship using the cost-of-carry model, indicating that a spot and its futures prices can diverge by no more than the convenience yield of the underlying commodity. In particular, WTI and Brent futures prices are actively used as references for valuing the underlying crude spots because short distant futures for WTI and Brent are highly liquid, and their prices can more effectively reflect market information for the underlying crudes. Such theoretical principles and pricing conventions pose the question of how the four-dimensional time-series – WTI spot, Brent spot, WTI futures, and Brent futures – are systematically related from a long-run movement perspective, and, if they are, how the relationships are persistent and consistent.

In this context, we scrutinize the interconnectedness of global benchmark crude oils and their financial markets using cointegration analysis. Cointegration refers to a long-run equilibrium relationship existing in two or more nonstationary time-series and was first proposed by the seminal works of Granger (1981) and Engle and Granger (1987). Cointegrated assets tend to have strong common patterns in long-term movement rather than short-term direction, which could be driven by the homogeneity of the underlying assets. Such comovements of cointegrated assets give rise to arbitrage opportunities, which have great implications for practitioners concerning trading strategies, portfolio allocation, and hedging schemes involving spots and futures.

The second concern is price discovery, which is the process of incorporating new information into security prices. For cointegrated processes, Figuerola-Ferretti and Gonzalo (2010) provide a theoretical basis that the permanent component represented in PT decomposition is strongly related to the price discovery process in terms of the information-share mechanism and attributable weight to provide price information. It has also been demonstrated that the PT decomposition and information share models are directly associated and provide similar results if the residuals of the VECM are uncorrelated between markets (Baillie et al., 2002). The adjusting parameter estimated in the VECM form is a straight measure of the contribution of each market to the price discovery process. In other words, this parameter measures how the previous period's disequilibrium error is fed into today's price dynamics.

Since the WTI and Brent crude markets are geographically fragmented, yet are contemporaneously influenced by global risk factors, it is a challenge to precisely detect how price discovery is established in the two crude markets. Moreover, since the financial markets of WTI and Brent tend to integrate because of the active engagement of arbitragers, it is worth examining which markets are taking price leadership between the two benchmark crude futures. This study has vital implications for pricing and forecasting (Kao and Wan, 2012; Elder et al., 2014). Thus, we test for detection of a component driving the price discovery mechanism, exclusively or even partially, in the cointegrated sequences. To do so, we apply PT decomposition analysis for all the cointegrated pairs possibly created by the two spots and two futures, by expanding the idea of the price discovery function to other relevant markets including WTI and Brent crudes, WTI spot, and Brent futures, and vice versa.

Our major empirical findings are twofold. First, the full-sample test shows that the four markets are entirely cointegrated, while the sub-sample test illustrates that patterns of the pairwise cointegration relationship are not only heterogeneous but also time-varying and non-linear. More specifically, the four prices remained highly cointegrated until 2010, but their overall connectedness tended to bifurcate into the WTI and Brent markets for the period from 2010 to 2019, and it finally turned to segregate to the spot versus futures markets for the recent half decade. Second, we evaluate the degree of the cointegration persistency for the four markets. The degree for WTI and Brent spot markets is at the strongest level, because their cointegration continued for 30 years, which is the longest among the six pairs. For the crude and its futures market pairs, the degree is the second highest, since their cointegration lasted for 26 years from 1993 to 2019 before being fragmented since 2019. However, the extent for the WTI and Brent futures markets is the lowest, as these markets have been disconnected for 16 years from 2003 to 2019 prior to being reunited since 2019. For the spot and futures cross pairs, the degree is in a moderate level, as their cointegration gradually weakened over time and finally disappeared.

Although the comovement propensity is vastly tested utilizing cointegration framework with price discovery analysis in a broad range of studies, to the best of our knowledge, there has been no in-depth investigation of the structure of market unitedness and price leadership among closely associated markets in terms of time-varying and systematic aspects. It is crucial to simultaneously monitor the cointegration tendencies in both spot and futures prices of WTI and Brent in practical marketplaces to achieve the best trading performance. This is because it is important for modern financial trading to utilize different types of commodity assets and derivatives for various purposes including pair-trading, cross-hedging, and spread options. For such activities to be more involved in markets can cause the asset dynamics to be comoving and synchronizing, implying that the uncertainty and dependency of the markets are susceptible to fluctuation, even due to a weak stimulus by an adverse event in one market. It can also be interpreted that such markets are prone to expose to systemic risk. Investigating price leadership and the extent of cointegration persistency in relevant markets can be a good guide for managing systemic risk from the perspective of institutions, traders, and regulators.

This study makes two contributions in this context. First, we demonstrate that a non-linear cointegration structure exists in the four-price dynamics with assessing price leadership via the full-sample and sub-sample tests. With this result, we illustrate the heterogeneous cointegration relationships in a chronological diagram to portray the 30-year history of the relationship structure in terms of unitedness and price supremacy for the crudes and their financial markets. Moreover, verifying that for the recent half decade, the WTI spot dominates the Brent spot in the crude market, while Brent futures leads WTI futures in the derivatives market, we can utilize to manage systemic risk in the respective markets from the perspective of long-term investors and regulators.

Second, we empirically validate the relevancy between cointegration and correlation. Although both measures can evaluate the dependency of time-indexed variables, they are differentiated in terms of definitions, and stochastic modelling schemes – correlation gauges dependency occurring in short-term movements, while cointegration focuses on detecting long-term common trends in the series. There is no exact theoretical relationship between the two measures, meaning that strongly (weakly) correlated time-series may be more likely to be (not) cointegrated. Nevertheless, their empirical consistency can be revealed, which is depending on the situation. As a result of comparing the results of correlation and cointegration in the sub-sample test, the coherent relevance between two values was not clearly observed. Although WTI and Brent spot prices have long persisted in cointegration at the highest level among the six pairs, their correlation is estimated to be the lowest for every time bucket. Meanwhile, despite WTI futures and Brent futures being cointegrated to the least degree, their correlation is the strongest.

The remainder of this paper is organized as follows: Section 2 reviews the literature in the areas covered by this study; Section 3 discusses the methodologies for measuring systemic risk; Section 4 presents the empirical results for the whole sample period by introducing the data sets; Section 5 presents the empirical results for the sub-sample sets; Section 6 provides implication from the empirical results of the whole versus the sub-sample periods; and Section 7 concludes the paper.

2 Literature Review

This study relates to prior research on cointegration analysis, which allows for multiple structural breaks based on the VECM approach and price discovery. This section reviews the literature on cointegration, when dealing with crude markets interpreted using PT decomposition.

Many financial data series are known to exhibit cointegration, for example, international stock markets (Cerchi and Havenner, 1988; Taylor and Tonks, 1989; Duan and Pliska, 2004), foreign exchange rates (Baillie and Bollerslev, 1989; Kellard et al., 2010), futures and spot prices (Ng and Pirrong, 1994, 1996; Maslyuka and Smyth, 2009). In particular, even though commodity prices are prone to frequent spikes and sudden drops, they show a sticky comovement propensity (e.g., crude oil, gasoline, and heating oil prices in Serletis, 1992; Chiu et al., 2015). WTI and Brent prices were studied to verify different views of integration versus fragmentation, or mixed, using cointegration analysis. Bentzen (2007) argues that international crude markets are united for testing Brent, Organization of the Petroleum Exporting Countries (OPEC), and WTI prices with high-frequency data, whereas Gulen (1997) tests whether the world oil market is unified if same quality crudes from different regions move together, and regionalized otherwise. Weiner (1991) shows that the world oil market has a high degree of regionalization, implying that it is far from completely unified.

VECM analysis is widely used to estimate the cointegration relationship developed by the pioneering works of Johansen (1988, 1995), which have been extended to diverse cases, including allowing a structural break in the deterministic trend (Johansen et al., 2002), threshold-embedded cointegration analysis (Balke and Fomby, 1997; Hammoudeh et al., 2008), and the fractional VECM model (Johansen and Nielsen, 2012; Mackinnon and Nielsen, 2014; Dolatabadi et al., 2015). The structural break model in the cointegration relationship was first studied in the primitive form of the cointegration equation (Fan and Xu, 2011; Noguera, 2013; Wang and Wu, 2013). These breaks occur because of economic or policy changes, wars or crises, or other significant events. Associated with the VECM representation, Gregory and Hansen (1996) and Hatemi-J (2008) propose an estimation method for respective, unique, and at most two, pre-fixed numbers of structural breaks at unknown time points, which was developed based on the structural break test for typical regression models proposed by Bai and Perron (1998) and the unit-root test with break-points by Kapetanios (2005). Maki (2012) generalizes the two previous studies to build a test for estimating where cointegration changes at a maximum of five unknown points.

Studies on price discovery have been documented in the extensive literature on various market types (Garbade and Silver, 1983; Stock and Watson, 1988; Hasbrouck, 1995; Baillie et al., 2002; Hasbrouck, 2021), which is rooted in the common factor model across a number of studies (see, Beveridge and Nelson, 1981; Quah, 1992; Stock and Watson, 1988, many others). The theoretical relationship between price discovery and PT decomposition has been also verified (Baillie et al., 2002; Figuerola-Ferretti and Gonzalo, 2010). Given this theoretical background, price discovery is vastly examined from an empirical perspective, focusing on the combination of a spot and its futures prices, as price discovery is considered an important function of trading futures contracts for pricing cash market transactions in commodity markets (Garbade and Silver, 1983; Figuerola-Ferretti and Gonzalo, 2010;

Dolatabadi et al., 2015; Yu et al., 2023 for commodities; and Wu et al., 2021 for bitcoins). For crude spot and its futures markets, PT analysis is employed to measure the extent of to contribution in price discovery between the two markets, incorporating convenience yield (Kao and Wan, 2012; Elder et al., 2014).

3 Methodology

Let the variables y_{it} be collected as a vector be $Y_t = (y_{1t}, \cdots, y_{nt})$. Assume that all components y_{it} are non-stationary, equivalently saying characterized by a unit root, that is, Y_t is integrated of order one, or $I(1)$. It implies the first difference of Y_t, or $\Delta Y_t = Y_t - Y_{t-1}$, is to be stationary, or $I(0)$. Such a vector Y_t is called *cointegrated* if there exists a vector $\beta = (\beta_1, \cdots, \beta_n)$ such that their linear combination $\beta' Y_t$ is stationary; and β is called a cointegration vector. Since the vector β is not unique, and $\beta_1 = 1$ can be set without loss of generality, finding a vector β that makes $y_{1t} + \beta_2 y_{2t} + \cdots + \beta_n y_{nt} = \epsilon_t$ for $\epsilon_t \sim I(0)$ implies that Y_t are cointegrated. It can be interpreted there exists the long-run equilibrium relation $\beta' Y_t = 0$ in an average sense, as ϵ_t has no effect to a long-term converging level.

Cointegration is closely connected to the VECM, which is investigated by a number of authors; see Stock (1987) and Johansen (1988) among others. In the VECM, the changes in a variable y_{it} depend on the deviations from the equilibrium relation $\beta' Y_t$. The VECM representation is given as follows:

$$\Delta Y_t = \Pi Y_{t-1} + \sum_{j=1}^{p} \phi_j \Delta Y_{t-j} + u_t, \tag{1}$$

where a matrix $\Pi \in \mathbb{R}^{n \times n}$, and u_t is a white noise error vector $(u_{1t}, ..., u_{nt})$. In Eq.(1), the summation term monitors the short-run dynamics of ΔY_t with order of p. Meanwhile, the term ΠY_{t-1} represents the degree of the long-run equilibrium, that is also known as the error correction term. If rank Π is r for $0 \leq r < n$, then r cointegration relationships exist and is decomposed into $\Pi = \alpha\beta'$, where $\alpha, \beta \in \mathbb{R}^{n \times r}$ with rank r. For example, the y-variables with cointegration rank one is present single equilibrium, so that $\Pi = \alpha\beta'$ with $\alpha = (\alpha_1, ..., \alpha_n)$; and hence ΠY_{t-1}, or equivalently, $\sum_{i=1}^{n} \beta_i y_{it-1}$ can be stationary. In the error correction term, $\beta' Y_{t-1}$ represents error from disequilibrium occurred in the previous step, while the vector α governs the adjustment speed reverting to the equilibrium level from the past disequilibrium. Without loss of generality, we may set $\tilde{Y}_t = (y_{2t}, y_{3t}, ..., y_{nt})$ and $\tilde{\beta} = (\beta_2, \beta_3, ..., \beta_n)$, equivalently, $Y_t = (y_{1t}, \tilde{Y}_t)$ and $\beta = (1, \tilde{\beta})$.

Our model considers a deterministic trend into cointegration relation such that

$$y_{1t} = \gamma_0 + \gamma_1 t - (\beta_2 y_{2t} + \cdots + \beta_n y_{nt}) + \epsilon_t, \tag{2}$$

where γ_0 is a constant level, and γ_1 is a constant time trend. If such a linear trend appears in the cointegration relation, the term ΠY_{t-1} in the VECM of Eq.(1) can be modified to

$$\Pi Y_{t-1} = \alpha(\beta' Y_{t-1} + \gamma_0 + \gamma_1 t),$$

where the additional deterministic term is interpreted as the mean level of the long-run equilibrium, i.e., $\mathbb{E}[\beta' Y_t] + \gamma_0 + \gamma_1 t = 0$.

Cointegration test is conducted by statistically inferencing the rank Π, which is equivalent to the number of cointegration relations $r < n$, for n variables. The trace statistic and the maximum eigenvalue statistic are commonly employed. The trace test is inferenced about the null hypothesis of r cointegrating relations against the alternative of n relations; while the maximum eigenvalue test concerns the null hypothesis of r cointegrating relations against $r+1$, for $r = 0, \cdots, n-1$. Both tests are developed based on the likelihood ratio (LR) of the null and alternative hypotheses and their asymptotic distribution (Johansen and Nielsen, 2012; Mackinnon and Nielsen, 2014; Dolatabadi et al., 2015). It turns out that both are to be slightly discrepant in small samples, yet the power of trace test is superior to that of maximum eigenvalue test in some situations (Lutkepohl et al., 2000).

Next, our interest is about a test of examining how a variable y_{it} contributes to build a long-run equilibrium, which reflects a common trend in the cointegrated series Y_t. This is commonly investigated by computing the orthogonal complements $\alpha_\perp$ of the adjusting vector α, where $\alpha_\perp$ is defined as $\alpha' \alpha_\perp = 0$. Alternatively, PT decomposition is also a method for detecting a contributor for the common trends linked to the VECM setup for multiple processes. Gonzalo and Granger (1995) demonstrate that linearity assumption of the common factor enables us to obtain the following unique representation: the element of the cointegrated vector Y_t with reduced rank r can be explained by $(n-r)$ common variables $I(1)$ plus transitory components $I(0)$. In other words, any cointegrated series Y_t can be decomposed into a permanent part represented by $f_t = \alpha_\perp Y_t$ and a transitory part by $z_t = \beta' Y_t$ such that

$$Y_t = A_1 f_t + A_2 z_t,$$

where $A_1 = \beta_\perp(\alpha'_\perp \beta_\perp)^{-1}$ and $A_2 = \alpha(\beta'\alpha)^{-1}$. The permanent component of the cointegrated variables plays a role of a dominant or leading price for the long-run equilibrium relationship, and the transitory components are of a follower for the common trend. This PT identification can be simply achieved by the error correction term estimate of α and β.

In addition, for a bivariate cointegrated series, we conduct the hypothesis tests on α. The interested hypothesis is

$$H^1_\alpha : \alpha_\perp = (1, 0),$$

which represents that price discovery is exclusively driven by the first variable. Equivalently, it can be tested with the mirror hypothesis $\alpha = (0, a)'$. Under which H^1_α is true, that is, $\alpha_1 = 0$, the first variable does not react to the disequilibrium error, indicating that the first variable is the solely main contributor to the price discovery and the second variable is only transitory. Similarly, to test if price discovery is exclusively present in the second variable,

$$H_\alpha^2 : \alpha_\perp = (0, 1),$$

or equivalently, the mirror hypothesis $\alpha = (a, 0)'$ can be tested. These hypothesis testing can be achieved using LR statistics within the framework of Johansen (1988), who computes critical values from chi-square distributions. The test statistics of H_α^1 and H_α^2 are given based on a chi-square distribution with one degree of freedom.

The previously discussed model assumes a linear and time-invariant cointegration relationship among variables. Recent econometric literature is interested in examining non-linear linkages for which cointegrated variables and its trends are rephasing with respect to time, especially for which crude oil markets are largely influenced by relevant politics, macroeconomy, and complicated geopolitical situations. In this regard, *structural breaks* are widely known as a major source of non-linearity in commodity markets, because such structural changes can be caused by extreme events in the associated markets, which can lead for regime shifts in price dynamic relations to occur.

To reflect the situations, we employ the model allowing jumps at unknown time points in the deterministic trend and cointegrating terms. We can estimate the structural breaks employing the Maki cointegration test (Maki, 2012), which assesses presence of cointegration relations by allowing multiple jumps in shift in level, time trend, and cointegration vector in Eq.(2) with assuming unknown number of jumps at the maximum k. The Maki model is describes as

$$y_{1t} = \gamma_0 + \sum_{i=1}^{k} \gamma_{0i}\delta_{i,t} + \gamma_1 t + \sum_{i=1}^{k} \gamma_{1i} t \delta_{i,t} + \tilde{\beta}' \tilde{Y}_t + \sum_{i=1}^{k} \tilde{\beta}_i' \tilde{Y}_t \delta_{i,t} + \epsilon_t, \tag{3}$$

where $\delta_{i,t} = 1_{(t \geq \tau_i)}$ is an indicator function for unknown multiple break date τ_i; and γ_{0i}, γ_{1i}, and $\tilde{\beta}_i$ represent a jump size at each break, for $i = 1, ..., k$. The Maki model enables us to ensure that such structural breaks are less likely to occur in testing existence of cointegration relationships for variables.

4 Empirical Analysis: Full-Sample Test

Our empirical analysis examines a bivariate cointegration relationship for the four variables. As displayed in Figure 1, we can obtain six pairs of variables, and the paired-variables are grouped as (a) the pair of two crude spots (futures) – WTI spot and Brent spot and WTI futures and Brent futures; (b) the pair of a spot and its futures – WTI spot and its futures, and Brent spot and its futures; and (c) the pair of a spot and the other crude's futures – WTI spot and Brent futures, and Brent spot and WTI futures.

For the six bivariate time-series, we conduct cointegration tests via a three-step procedure. First, we perform a full-sample test to examine linear cointegration relationships by applying the VECM to the entire time-period. Next, we implement the Maki model (3) for the four time-series on the whole time-period data to find possible structural break dates for them. We finally perform a sub-sample test by revisiting the VECM-based cointegration tests on the sliced data set with respect to the break dates.

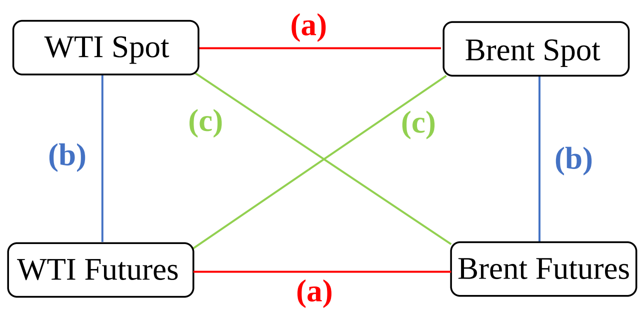

Figure 1: A diagram of the pairs of variables considered in our empirical test: (a) two crude spots and two crude futures, (b) a spot and the corresponding futures, (c) a spot and the cross spot's futures.

4.1 Data Description

WTI crude futures have been traded on the New York Mercantile Exchange (NYMEX) since 1983, which was developed to manage the volatility of crude prices after the US oil price decontrol. Brent futures, however, started publicly trading with open outcry system in 1988, transitioning to an electronic system on the Intercontinental Exchange (ICE) in 2005. Both futures contracts have similar specifications, where the unit contract size is 1,000 barrels, contracts are traded in US dollars with minimum price fluctuation of one cent and are cash-settled, and so on.

We choose daily closing prices for WTI spot, Brent spot, WTI 3-month futures, and Brent 3-month futures[2], where the spot and the futures prices are obtained from the Energy

[2] WTI and Brent futures are also traded in ICE and NYMEX, respectively, where WTI futures traded

Table 1: Summary statistics of WTI and Brent crude oil spot and futures log returns over the full-sample period of 1993 to 2022

	WTI Spot	WTI Fut	Brent Spot	Brent Fut
Mean	0.000	0.000	0.000	0.000
Median	0.001	0.001	0.001	0.001
Maximum	0.426	0.200	0.412	0.135
Minimum	-0.720	-0.454	-0.773	-0.312
Standard deviation	0.028	0.023	0.026	0.021
Skewness	-1.856	-1.595	-3.135	-0.804
Kurtosis	81.814	34.640	122.926	16.266

Information Administration [3] and the Quandle database[4], respectively. Since both futures contracts prices are available to the public after 1988, we consider it as the test period when reaching a proper level of liquidity and trade stabilization from the initial launch in the public markets. Our test period spans from October 1, 1993 to June 27, 2022, yielding 7,109 observations. All prices are measured in US dollars per barrel, and the data from diverse source match exactly with respect to the date.

For the cointegration tests, we convert the variables to log scale such that $Y_t = \ln X_t$ and $\varDelta Y_t = \ln X_t - \ln X_{t-1}$ representing the daily log return for the observed value X_t. This allows us to interpret the cointegration vector estimated as elasticities of the prices. Table 1 presents the summary statistics for the daily log returns for the four variables over the period from 1993 to 2022. The means are estimated very close to zero, and the standard deviations lie between 2% to 3% level. Both crude prices have bigger standard deviation than corresponding futures prices. All daily returns are negatively skewed, where skewness of the Brent spot is the largest among the four. Kurtosis of the returns ranges diverse with the Brent spot having the largest level. The returns are far from a normal distribution, largely skewed and peaked. The non-normality facets are seen via kernel density estimation, as displayed in Figure 2, also verified using the Jarque-Bera and Kolmogorov-Smirnov tests.

Figure 3 displays the daily log prices for the obtained sample sets and the original price spreads of Brent-WTI spots and Brent-WTI futures, respectively, over the test period. Even though WTI and Brent spot prices have been closely tracked as a whole, yet as seen the price difference measured by the Brent-WTI spread, it fluctuates greatly over time,

in CME accounts for 80%, whereas that in ICE accounts for 20%; and the Brent futures in ICE accounts for 90%, whereas that in NYMEX accounts for 10%. The same futures traded in the different exchanges are priced identical.

[3] https://www.eia.gov/dnav/pet/pet_pri_spt_s1_d.htm

[4] Quandl provides the futures data delivered from CME and ICE as a form of continuously connected time-series. The futures data chosen has two to three months by maturity.

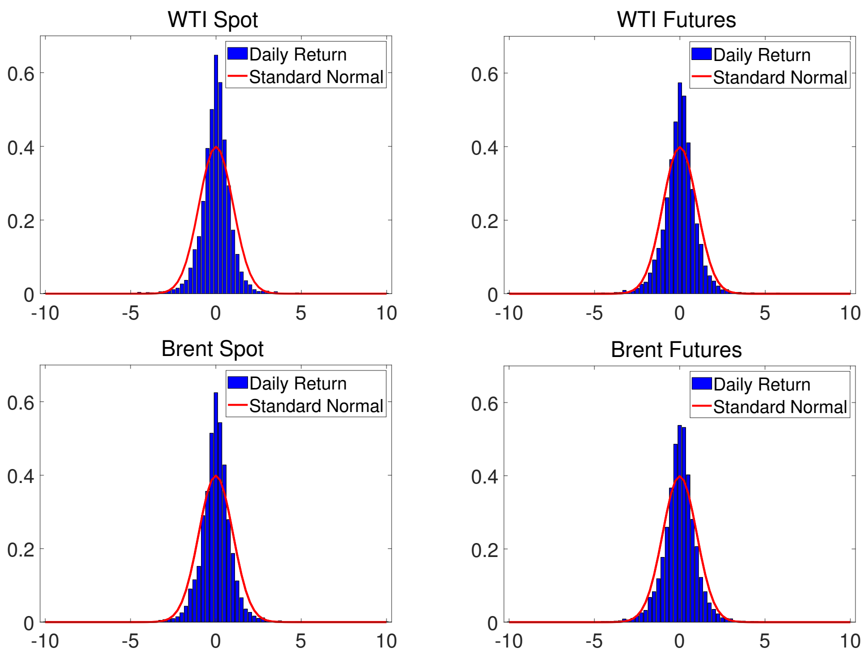

Figure 2: Kernel density estimation of the daily log returns with a standard normal density for WTI and Brent crude oil spot and futures on the full-sample period of 1993 to 2022

reflecting various market factors, including energy policy, supply/demand conditions, and inventory levels (Fattouh, 2010; Liu et al., 2018). There is present one notable regime change in the spot spread's long-term trend. The spread remained negative for a long time, but in early 2011 it became large positive, which trend has been lasting so far. The premium of WTI over Brent before 2011 was due to the better quality of WTI crude over Brent and for WTI globally less supplied due to the US crude export-ban policy. The discount on WTI against Brent since 2011 is viewed mainly as a result of lifting the export-ban policy and the surge in the shale oil production in the US (Melk and Ojeda, 2014; Caporin et al., 2019). In our data set, the Brent-WTI spread is -\$1.4 on average until the end of 2010 from the beginning of the sample period, but it dramatically widened \$6.9 on average since then. In particular, the spread rose \$30 reaching at ever highest in September 2011.

The futures spread exhibits a very similar pattern with their spot dynamics yet less volatile, where standard deviations for the spread of the spots and the futures are \$5.9 and \$5.2, respectively. One remarkable event is the WTI future May contract price plunged negative on 20 April 2020 for the first time in history, dropping by -306% in a day and trading closed at -\$37.63. It happened due to an unprecedented global energy glut as the Covid pandemic halts travel and curbs economic activity (Fernandez-Perez et al., 2023).

Table 2: The p values of ADF (left) and PP (right) tests are reported applying for the log price level Y_t (upper) and the log price first difference ΔY_t (lower) for the full-sample. The null hypothesis for both tests is the presence of a unit root.

	WTI Spot	WTI Fut	Brent Spot	Brent Fut	WTI Spot	WTI Fut	Brent Spot	Brent Fut
	ADF				PP			
Y_t	0.316	0.513	0.321	0.542	0.403	0.596	0.469	0.677
	ADF				PP			
ΔY_t	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

Note: The p value smaller than 0.01 is reported to 0.01.

The prices recovered the next day to $10.01 per barrel. WTI futures contracts have remained above zero since, having recovered more than half of the pre-Covid pandemic price.

4.2 Results

As the cointegration models are described in the previous section, we need to confirm that Y_t is $I(1)$ and ΔY_t is $I(0)$, prior to the cointegration test. To deal with the issue, we conduct a unit-root test based on the augmented Dicky-Fuller (ADF) and Phillips-Perron (PP) tests, where both null hypothesize presence of a unit-root, that is, non-stationarity for the series of interest. Table 2 reports the p values of the null hypothesis of the presence of a unit root in Y_t and ΔY_t for each series. At the 1% significance level, the null hypothesis cannot be rejected for Y_t in the ADF and PP tests, whereas it is rejected for ΔY_t for the four series. This confirms that the crude oil prices and their futures are $I(1)$.

We then implement rank test of the cointegrating vector using the trace test with LR statistic, following the numerical procedure by Nielsen and Popiel (2018). Assuming a full rank $r = n$, we first determine the lag length k by testing the null hypothesis $\phi_j = 0$ based on LR statistic adding to the Akaike and Bayesian information criterion. For the individual residual series the tests of white noise and Ljung-Box serial correlation with 10 lags are also conducted up to order (Dolatabadi et al., 2015). Once the lag-order is chosen, the cointegration rank is tested based on LR statistic for the null hypothesis of the existence of at most $r \leq n$ cointegrating relations against a full rank for each pair. With the rank and lag selected, the main parameters are estimated including α, β, ϕ_j's, and the deterministic components.

Table 3 shows the result of the bivariate cointegration rank test and the error correction term estimation for the whole sample period. The variable in the first line is used as a dependent variable. Panel A shows that one cointegrating vector exists as the hypothesis $r = 0$ is rejected against $r = 2$, while $r = 1$ is not rejected, for all six pairs. We can confirm

Table 3: Cointegration rank test and bivariate VECM estimation of the full-sample for each pair. The p values of LR statistic in Panel A, the estimates of β, α with the corresponding standard errors (in parentheses) in Panel B, and the orthogonal components of $\alpha_\perp$ and the p values for H_α^i with the ones gray-colored representing inconclusive case in Panel C.

		(a)		(b)		(c)	
	y_1	WTI Spot	WTI Fut	WTI Spot	Brent Spot	WTI Spot	Brent Spot
	y_2	Brent Spot	Brent Fut	WTI Fut	Brent Fut	Brent Fut	WTI Fut
Panel A	$r = 0$	0.000	0.001	0.000	0.000	0.000	0.000
	$r = 1$	0.494	0.576	0.543	0.575	0.584	0.545
Panel B	β	-0.905	-0.907	-0.992	-0.998	-0.899	-1.093
	α_1	-0.024	-0.008	-0.039	-0.035	-0.019	-0.016
	(s.e.)	(0.006)	(0.005)	(0.008)	(0.006)	(0.005)	(0.004)
	α_2	0.012	0.000	0.005	0.006	0.001	0.006
	(s.e.)	(0.005)	(0.005)	(0.007)	(0.005)	(0.004)	(0.004)
Panel C	$\alpha_{1\perp}$	0.333	0.027	0.106	0.138	0.046	0.256
	$\alpha_{2\perp}$	0.667	0.973	0.894	0.862	0.954	0.744
	H_α^1	0.000***	0.000***	0.000***	0.000***	0.001***	0.173
	H_α^2	0.497	0.026**	0.807	0.317	0.181	0.968

cointegration relationships exist for the six pairs. Panel B exhibits the estimates for β, α with standard error in parentheses, and Panel C shows the estimates of the orthogonal vector $\alpha_\perp$ and the p values for the hypothesis H_α^i at the 1%, 5%, and 10% levels, and the gray-colored cell represents the case of inconclusive, where both are rejected at the same significance levels, or neither are rejected on the mutually exclusive hypotheses. The value $|\beta| < 1$ for the spot and future pair shows long-run contango in the WTI (Brent) spot and its futures prices. The results of PT decomposition in Panel C show the followings: Brent is a major contributor to long-run equilibrium over WTI in both spot and futures markets (see case (a)). The futures price dominates its spot price in long-run relationship for both WTI and Brent (see case (b)), although the cross-pair cases are inconclusive (see case (c)).

5 Empirical Analysis: Sub-Sample Test

This section discusses a sub-sample test. Applying the Maki cointegration test to the entire sample set to check out possible presence of structural breaks for the four series to be cointegrated, we perform the cointegration rank test and the PT decomposition analysis on the sub-sample periods confirming that structural changes are less likely to occur for each period.

Table 4: The result of the Maki cointegration test with the test statistics and the critical values at 1%, 5%, 10% allowing the maximum number of structural break points from 1 to 5 and the corresponding estimated dates

Number of Breaks	Test Statistics	Critical Values at 1%, 5%, 10%	H_0 No Cointegration	Estimated Date
1	-10.95	-6.70, -6.46, -6.22	Not Reject	2011/01/13
2	-10.95	-7.74, -7.20, -6.93	Not Reject	2011/01/13, 2019/06/27
3	-11.18	-8.33, -7.74, -7.45	Not Reject	2002/12/24, 2011/01/13, 2019/06/27
4	-11.47	-8.85, -8.27, -7.96	Not Reject	2002/12/24, 2011/01/13, 2015/12/14, 2019/06/27
5	-11.72^{***}	-9.42, 8.80, -8.51	Reject	2002/12/24, 2007/7/25, 2011/1/13, 2015/12/14, 2019/06/27

5.1 Maki Cointegration Test

We implement the Maki model (3) for the four variables by setting the WTI spot price as a dependent variable (y) and the other three variables as an independent variable (x_i). We conduct the four-variate Maki test on the null hypothesis of no cointegration with allowing the maximum number of possible break points from 1 to 5 in the trend and regime shift terms. We obtain five structural breaks because the test rejects the hypothesis of no cointegration in the four series at the 1% level with five break points, whereas the tests for the other four cases cannot reject at any confidence level. The corresponding estimated break dates are obtained as December 24, 2002; July 25, 2007; January 13, 2011; December 14, 2015; and June 27, 2019. Table 4 shows the Maki test result for the five cases of the number of break points and the corresponding estimated break date(s), where the critical values at the 1%, 5%, and 10% levels and the calculated test statistic value are described for each case.

Table 5 documents the summary statistics for the daily log returns computed on each sub-sample period divided, which spans a landscape for the international benchmark crude markets for the past 30 years. During the whole test period, the markets exhibited relatively stable and calm although global economic and geopolitical situations were unstable having incurred the Gulf war in 1990, the Asian economic crisis in 1998, and the 911 attacks in 2001. For 2003 to 2007, the markets rose steeply due to not only continuing of unstable global situation including the US's Iraq invasion and Hurricanes Dennis, Emily, and Katrina, but also dramatic increase of crude's demands by emerging Asian countries including China and India. The period 2007 - 2011 underwent a large drop for a very short period due to the global financial crisis, that is the worst ever economic recession, resulting in that inventory stock builds and the leaving markets more highly volatile than ever.

During 2011 to 2015, the Brent-WTI spread turned to be positive, and largely widened indicating that the two crude markets started segregating. The crude markets experienced another plummet influenced by the shale oil revolution and relevant policies changes. During 2015 to 2019, the markets slightly recovered, where a price-soaring occurred in 2016 when OPEC's decision to cut production quota caused speculative activities, and the 40-year lasting export-ban on US crude was lifted in 2015 causing the price to fall and the price volatility to rise. For 2019 to 2022, the markets encountered prominent price-tumbling and volatility-rising abruptly in early 2020 due to the outbreak of Covid pandemic. During the Covid era the WTI price ran into negative price in 2021 because of global lockdowns and travel restrictions, and also it was exacerbated by the Russia-Saudi Arabia oil price war in 2020 caused an unprecedented sharp fall in demand (Miller et al., 2010; Le et al., 2021, 2023).

We examine the time-varying volatility for the four series and correlation for the six paired series by adopting the exponential weighted moving average (EWMA) method. Figure 4 illustrates the one-day EWMA volatility series for each series and the pair-wise EWMA correlation series using a weighting rate of 0.94, where the horizontal line represents a structural break point. Figures 5 and 4 display the averaged one-day EWMA volatility and the pair-wise correlation on each period based on daily values, respectively. For the volatility, the figures stayed oscillating within a bounded range, but they soared during the global credit crisis in 2008 and skyrocketed at the outbreak of Covid in early 2020. This fact is also seen in the averaged volatilities in Figure 5, where relatively high volatilities appear in periods 3 and 6. An interesting fact is that there exists a propensity that the volatilities of the crude spot prices are higher than those of the futures prices for most of the period. Such a differential between the crudes and their futures markets emerged more evidently in the first and last periods, as can be seen in the averaged values in Figure 5.

For the correlation, WTI and Brent spot prices are the most loosely correlated at about the 60% level, whereas WTI and Brent futures prices always remain at the highest over 95% except for period 4, which fell to 88%, which is the time when the Brent-WTI spot and futures spread widened substantially. When the Covid outbreak occurred, the correlation between the two crude prices plummeted to 14% temporarily, but it reverted to the previous level. A notable observation is that the Brent spot-involved pairs engaged with WTI spot, WTI futures, and Brent spot exhibit relatively less correlated than the other three pairs. The evident discrepancies between the two groups are shown in Figure 5. Correlations of WTI futures with the other three prices ranges around 60% on average, whereas the other pairs remain at a much higher level over 80%. This gap, however, tended to gradually narrow over time because correlations of the Brent spot involved pairs has significantly increased in the recent decades.

Table 5: Summary statistics for the daily log returns of WTI spot, its futures, Brent spot, and its futures on the six periods of October 1993 to December 2002 (right, top); January 2003 to July 2007 (left, top); August 2007 to January 2011 (right, middle); February 2011 to December 2015 (left, middle); January 2016 to June 2019 (right, bottom); and July 2019 to June 2022 (left, bottom).

	WTI Spot	WTI Fut	Brent Spot	Brent Fut	WTI Spot	WTI Fut	Brent Spot	Brent Fut
	1993 - 2002				2003 - 2007			
Mean	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001
Median	0.001	0.001	0.001	0.000	0.001	0.001	0.001	0.001
Maximum	0.154	0.114	0.163	0.116	0.124	0.059	0.115	0.060
Minimum	-0.171	-0.157	-0.199	-0.139	-0.152	-0.077	-0.090	-0.079
Standard deviation	0.024	0.019	0.024	0.019	0.023	0.019	0.021	0.019
Skewness	-0.292	-0.399	-0.182	-0.270	-0.426	-0.173	-0.172	-0.117
Kurtosis	7.559	7.394	7.895	7.150	6.844	3.576	4.410	3.643
	2007 - 2011				2011 - 2015			
Mean	0.000	0.000	0.000	0.000	-0.001	-0.001	-0.001	-0.001
Median	0.000	0.001	0.001	0.001	0.000	0.000	0.000	0.000
Maximum	0.164	0.128	0.181	0.129	0.098	0.097	0.085	0.096
Minimum	-0.128	-0.104	-0.168	-0.096	-0.111	-0.107	-0.082	-0.102
Standard deviation	0.030	0.026	0.027	0.025	0.020	0.019	0.017	0.017
Skewness	0.125	-0.083	0.068	-0.044	-0.123	-0.157	-0.117	-0.230
Kurtosis	6.976	5.332	8.599	5.304	6.239	6.393	6.392	7.132
	2016 - 2019				2019 - 2022			
Mean	0.001	0.000	0.001	0.001	0.001	0.001	0.001	0.001
Median	0.001	0.002	0.001	0.002	0.002	0.003	0.003	0.003
Maximum	0.113	0.096	0.099	0.099	0.426	0.200	0.412	0.135
Minimum	-0.080	-0.074	-0.064	-0.081	-0.720	-0.454	-0.773	-0.312
Standard deviation	0.022	0.020	0.022	0.020	0.053	0.036	0.049	0.031
Skewness	0.203	-0.037	0.315	0.027	-2.719	-3.388	-4.641	-2.265
Kurtosis	5.556	4.756	5.093	5.465	63.621	44.126	95.225	26.402

Table 6: The p values of ADF (left) and PP (right) tests are reported applying for the log price level Y_t (upper) and it first difference ΔY_t (lower) on the six periods.

	WTI Spot	WTI Fut	Brent Spot	Brent Fut	WTI Spot	WTI Fut	Brent Spot	Brent Fut
Y_t	ADF				PP			
1993 - 2002	0.551	0.531	0.404	0.530	0.406	0.515	0.460	0.592
2003 - 2007	0.368	0.466	0.188	0.462	0.241	0.471	0.186	0.490
2007 - 2011	0.905	0.888	0.920	0.917	0.844	0.881	0.913	0.913
2011 - 2015	0.843	0.913	0.914	0.945	0.915	0.927	0.921	0.957
2016 - 2019	0.126	0.091	0.069	0.094	0.432	0.403	0.340	0.425
2019 - 2022	0.316	0.513	0.321	0.542	0.403	0.596	0.469	0.677
ΔY_t	ADF				PP			
1993 - 2002	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
2003 - 2007	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
2007 - 2011	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
2011 - 2015	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
2016 - 2019	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
2019 - 2022	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

Note: The p values for log returns are smaller than the reported 0.01. The null hypotheses for both tests are the presence of a unit root.

5.2 Results

With the five structural breaks obtained from the Maki test, we implement the same analysis taken to the full-sample period to the six sub-sample periods, including the stationary test, the bivariate cointegration test, and the PT decomposition. The numbers of the observations for the six sub-sample periods are 2258, 1129, 882, 1232, 870, and 738, in time order, where each number of samples is sufficiently enough to run the cointegration test. As a priori test, the ADF and PP tests are carried out for 24 time-series in total. Table 6 reports the p values of the ADF and PP tests to check out if $Y_t \sim I(1)$ and $\Delta Y_t \sim I(0)$, for each case, confirming that the 24 sub-sample time-series are $I(1)$.

Again, we perform the bivariate cointegration rank test based on the trace LR statistic after taking lag-order selection from the first to the sixth sub-sample period. Table 7 shows the respective p values for the cointegration rank tests of the null hypotheses $r = 0$ and $r = 1$ against $r = 2$ at the 5% level. The gray-colored cells illustrate the case when the null hypothesis $r = 0$ is not rejected, which means that no cointegration is present. We can see that the WTI and Brent futures had not been cointegrated for the period 2003 to 2019, and the cointegration relation was lifted during 2019 to 2022 for cases (b) and (c).

We finally estimate the values β and α in a bivariate VECM with the orthogonal vector $\alpha_\perp$, and also test H^i_α to verify the dominance variable on each period. Tables 8, 9, and

Table 7: Cointegration rank tests for the six sub-sample sets. The p values are reported for the six pairs and on the six sub-sample periods.

H_0	1993 - 2002	2003 - 2007	2007 - 2011	2011 - 2015	2016 - 2019	2019 - 2022
			(a) WTI Spot (y) & Brent Spot (x_1)			
$r = 0$	0.000	0.000	0.000	0.028	0.013	0.000
$r = 1$	0.519	0.710	0.814	0.103	0.066	0.916
			(a) WTI Fut & Brent Fut			
$r = 0$	0.000	0.543	0.487	0.100	0.304	0.006
$r = 1$	0.714	0.322	0.838	0.093	0.768	0.975
			(b) WTI Spot & WTI Fut			
$r = 0$	0.000	0.000	0.000	0.000	0.006	0.104
$r = 1$	0.550	0.631	0.745	0.556	0.677	0.599
			(b) Brent Spot & Brent Fut			
$r = 0$	0.000	0.000	0.004	0.000	0.000	0.215
$r = 1$	0.587	0.623	0.549	0.256	0.342	0.968
			(c) WTI Spot & Brent Fut			
$r = 0$	0.000	0.000	0.001	0.067	0.030	0.084
$r = 1$	0.636	0.579	0.805	0.070	0.112	0.584
			(c) Brent Spot & WTI Fut			
$r = 0$	0.000	0.000	0.011	0.047	0.102	0.147
$r = 1$	0.566	0.660	0.734	0.115	0.190	0.631

10 exhibit the results for cases (a), (b), and (c), respectively; and the gray-colored cell represents an inconclusive case for the mutually exclusive hypotheses H_α^1 and H_α^2.

The case (a) shows that the WTI and Brent spots are cointegrated in all the six sub-sample periods, where the WTI spot dominates a cointegration relationship during 2003 to 2019, yet the leading power is changed to the Brent spot since 2019. The WTI and Brent futures exhibit cointegration only for the periods 1993 to 2002 and 2019 to 2022, where the WTI futures dominates its relationship from 2019 to 2022. From 1993 to 2002, neither pairs of the two spots nor two futures is determined as a major contributor.

The case (b) shows the WTI spot and futures are cointegrated for the period from 1993 to 2019. There is not present a consistent pattern for determining the relationship leading factor. The similar pattern is observed in the Brent spot and futures. Although the patterns of long-run contango and backwardation have been changing over time, the contango occurred in both WTI and Brent during 2003 to 2007 and 2016 to 2019, and the backwardation occurred in both WTI and Brent for the other period.

The case (c) shows the leading factor with the previous cases is not consistently determined to one side. During 1993 to 2002, no component in the two pairs is determined to be a major contributor to cointegration.

Table 8: Case (a): The bivariate VECM estimates of β, α, $\alpha_\perp$, and the p value of H_α for the WTI and Brent spots (top) and WTI and Brent futures prices (bottom) on the six sub-sample periods.

	1993 - 2002	2003 - 2007	2007 - 2011	2011 - 2015	2016 - 2019	2019 - 2022
			WTI Spot & Brent Spot			
β	-0.917	-0.902	-1.011	-0.972	-0.716	-0.988
α_1	-0.073	-0.099	-0.106	-0.023	-0.050	-0.134
α_2	0.056	0.022	0.034	-0.001	-0.023	0.135
$\alpha_{1\perp}$	0.435	0.183	0.242	-0.068	-0.838	0.502
$\alpha_{2\perp}$	0.565	0.817	0.758	1.068	1.838	0.498
H_α^1	0.000***	0.000***	0.000***	0.022**	0.011**	0.016
H_α^2	0.000***	0.268	0.128	0.863	0.219	0.008***
			WTI Fut & Brent Futures			
β	-0.924	-	-	-	-	-1.040
α_1	-0.025	-	-	-	-	-0.118
α_2	0.021	-	-	-	-	-0.038
$\alpha_{1\perp}$	0.447	-	-	-	-	-0.471
$\alpha_{2\perp}$	0.553	-	-	-	-	1.471
H_α^1	0.327	-	-	-	-	0.064*
H_α^2	0.425	-	-	-	-	0.484

Table 9: Case (b): The bivariate VECM estimates of β, α, $\alpha_\perp$, and the p value of H_α for the WTI spot and WTI futures (top) and Brent spot and Brent futures prices (bottom) on the six sub-sample periods.

	1993 - 2002	2003 - 2007	2007 - 2011	2011 - 2015	2016 - 2019	2019 - 2022
			WTI Spot & WTI Futures			
β	-1.059	-0.924	-1.108	-1.049	-0.998	-
α_1	-0.025	-0.143	-0.013	-0.063	-0.094	-
α_2	0.034	-0.018	0.084	-0.005	-0.053	-
$\alpha_{1\perp}$	0.581	-0.146	0.866	-0.093	-1.284	-
$\alpha_{2\perp}$	0.419	1.146	0.134	1.093	2.284	-
H_α^1	0.119	0.000***	0.706	0.166	0.001***	-
H_α^2	0.005***	0.505	0.004***	0.902	0.045**	-
			Brent Spot & Brent Futures			
β	-1.065	-0.936	-1.090	-1.066	-0.997	-
α_1	-0.029	-0.075	0.005	-0.089	-0.097	-
α_2	0.025	0.028	0.101	0.049	-0.020	-
$\alpha_{1\perp}$	0.456	0.269	1.050	0.352	-0.268	-
$\alpha_{2\perp}$	0.544	0.731	-0.050	0.648	1.268	-
H_α^1	0.006***	0.001***	0.876	0.005***	0.000***	-
H_α^2	0.007***	0.196	0.002***	0.161	0.417	-

Table 10: Case (c): The bivariate VECM estimates of β, α, $\alpha_\perp$, and the p value of H_α for the WTI spot and Brent futures (top) and Brent spot and WTI futures prices (bottom) on the six sub-sample periods.

	1993 - 2002	2003 - 2007	2007 - 2011	2011 - 2015	2016 - 2019	2019 - 2022
			WTI Spot & Brent Futures			
β	-0.974	-0.843	-1.096	-	-0.820	-
α_1	-0.024	-0.079	-0.022	-	-0.052	-
α_2	0.022	-0.013	0.039	-	-0.028	-
$\alpha_{1\perp}$	0.481	-0.200	0.645	-	-1.158	-
$\alpha_{2\perp}$	0.519	1.200	0.355	-	2.158	-
H_α^1	0.050**	0.000***	0.380	-	0.031**	-
H_α^2	0.018**	0.478	0.055*	-	0.202	-
			Brent Spot & WTI Futures			
β	-1.159	-1.023	-1.105	-1.028	-	-
α_1	-0.033	-0.052	0.019	0.003	-	-
α_2	0.027	0.032	0.105	0.021	-	-
$\alpha_{1\perp}$	0.445	0.380	1.228	1.194	-	-
$\alpha_{2\perp}$	0.555	0.620	-0.228	-0.194	-	-
H_α^1	0.002***	0.007***	0.472	0.679	-	-
H_α^2	0.005***	0.089*	0.000***	0.026**	-	-

6 Implication

The sequential tests discussed in the previous sections demonstrated how the spot and futures markets for two benchmark crudes WTI and Brent are related in different aspects: cointegration, its persistency, and price discovery. In this section we recall and rearrange the preceding test results to draw several interpretations utilizing graphical charts and diagrams.

Figure 6 exhibits a diagram for the six pairs on the full-sample period, presence of cointegration is illustrated as a black line, while no presence of cointegration as a gray line. Also, the estimate of $|\beta|$ adjusted less than one[5] is displayed beside the cointegration line, which is to indicate how the price levels are close, noting that $|\beta| = 1$ means absolute distance between both processes are close to zero. This diagram also contains the dominant component between two prices with an arrow direction; for example, 'WTI spot $\rightarrow$ Brent spot' means the Brent spot price dominantly contributes to the equilibrium. The dotted line represents the case that the dominant factor is not clearly determined as one of two prices. Figure 7 shows the results of the sub-sample tests with which the same illustration rules with Figure 6 are applied.

[5]For the case that the estimate is greater than one, its reciprocal is taken.

The full-sample result shows the six pairs are cointegrated. Judging the magnitude of $|\beta|$, the highest similarity in price dynamics appears in the Brent spot and its futures, whereas the lowest one is in the WTI spot and Brent futures. It also manifests that the futures are leading its underlying and cross spot prices toward the equilibrium, and the Brent markets are leading WTI markets toward own equilibrium.

The sub-sample test result illustrates that, as a whole, the four markets remained relatively strongly cointegrated until 2010, but their entire connectedness tended to bifurcate into the WTI and Brent markets weakly during 2010 to 2019, and it finally turned to segregate to the spot versus futures markets in recent days. More specifically, in the period 1, all the six pairs stay cointegrated, but the leading component is inconclusive, except for the case of WTI spot and futures. In the period 2, the WTI and Brent futures become separated. Except for this case, the other five pairs remain cointegrated and its dominating factor is clearly determined, where the Brent spot leads the WTI spot, and the futures prices are mostly in a dominant position. During the period 3, the cointegration pattern like period 2 is present, however the leading element is changed to the spot price. In the period 4, cointegration of the WTI spot and Brent futures becomes fragmented though, the other four pairs remain united. The Brent spot keeps taking a leading position with the other associated prices, whereas the lead-follow components for the WTI spot and its futures becomes indistinct. In the period 5, cointegration relation continues to linger between the two crudes and between the crude and its futures, yet the leading position is reverted to the futures. The WTI spot and Brent futures pair come reunited, whereas the Brent spot and WTI futures pair gets divorced. In the period 6, the long-standing cointegration relationship between the spot and futures markets (WTI spot and its futures, Brent spot and its futures, and cross pairs) disappears completely. The WTI and Brent crude spots remain in cointegration, where the WTI spot takes a leading position over the Brent spot. The WTI and Brent futures become reconnected since 2003.

The sub-sample results enable us to evaluate the degree of the persistency of the cointegration relationship for the four time-series. (i) The persistency of the long-run relationship between the WTI and Brent spot markets is assessed at the highest level, as their cointegration relationship lasted for 30 years, all the sub-sample periods, which is the longest among the six pairs. (ii) The cointegration relationship between the crude and its futures markets is persistent at the second highest extent since it lasted for 26 years from 1993 to 2019 before fragmented since 2019. On the other hand, the cointegration between the two futures markets persists at the lowest degree, as these markets remained disconnected for 16 years from 2003 to 2019 yet became reunited since 2019, which is the shortest among the six pairs. (iii) The persistency of the cointegration relationships between the two cross-pairs is assessed in a moderate level, as it has gradually weakened over time and finally disappeared.

For the PT analysis, we can observe that the (WTI and Brent) futures prices tend to take a price leading position, whereas their underlying crude spot prices tend to take a following position. For the recent half decade, the WTI spot dominates the Brent spot in the crude market, while Brent futures leads WTI futures in the derivatives market. In terms of systemic risk, it can be more efficient to monitor the WTI crude and the Brent futures prices to control systemic risk, rather than its counterpart. Long-term viewers such as pension fund traders, institutions, and regulators can utilize these empirical facts to manage systemic risk more effectively.

Another interesting observation from our empirical tests is about comparison between correlation and cointegration. Correlation measures linear dependency appearing in the short-run fluctuating terms for stationary series associated with their volatility. Cointegration focuses on capturing common stochastic trends for non-stationary dynamics emerging in a long-run sense. Since these two measurements have totally different viewpoints for dealing with dependency in an econometric sense, there exists no exact theoretical relationship between them, which means that strongly (weakly) correlated time-series may be more likely to be (not) cointegrated, or vice versa. Nevertheless, the empirical consistency between the two concepts can be revealed, which should be depending on the situations. To test if this coherent relevance between the two measures is present in our data set, we compare the results between the one-day EWMA correlation averaged for each period in Figure 5 and the sub-sample cointegration test for the four series. We can tell that the coherent relationship is not clearly shown. Although WTI and Brent spot prices have the longest persisted in cointegration among the six pairs, their correlation is computed as the lowest level. Meanwhile, despite of WTI futures and Brent futures are cointegrated during the least period among the six pairs, their correlation is estimated to be the strongest.

For a short-term trader's view including proprietary traders, hedgers, and high-frequency traders, monitoring and utilizing correlation may be more suitable for their trading purposes, while for an agent needing a long-term vision such as pension fund managers and regulators, judging the cointegration of the markets can be more informative.

7 Conclusion

The purpose of this study is to examine the persistency of the long-run relationship in four time-series of the spot and futures prices of WTI and Brent crudes using cointegration analysis and to learn the price leadership in them over time. To do so, we employ the VECM approach and Maki cointegration test to detect the structural breaks and evaluate the cointegration relationship among the four time-series. We also conduct the PT decomposition analysis to verify the price leadership in the four asset dynamics. Data are collected with daily prices from 1993 to 2022, that are divided into the six sub-sample periods by the five structural breaks employing the Maki's regime and trend shift model.

For the six pairs of the four time-series, we proceeded the pairwise cointegration test via the three-step procedure: the bivariate VECM test and the PT analysis for the full-sample, the four-variate Maki test to estimate the structural breaks in the whole period, and the bivariate VECM and PT analysis on the six sub-sample periods. Through the sequential tests, we discovered that the four markets are overall cointegrated for the full-sample period, yet the cointegration patterns are not only inhomogeneous with respect to a pair of variables, but also time-varying and non-linear with respect to time change. The full-sample test showed that the futures markets dominate the spot markets, judging by the PT results for the six pairs. The sub-sample test verified that the four prices remain relatively highly cointegrated until 2010, whereas they begin fragmenting into two groups, the WTI and Brent markets, from 2010 to 2019, and finally segregate into the spot and futures markets recently. The PT results illustrated that the WTI spot dominantly contributes to the cointegration relationship over the Brent spot, while the Brent futures leads the relationship over WTI futures.

We also evaluated the degree of the persistency of cointegration for the four markets. The WTI and Brent spot markets appeared at the strongest level, because its cointegration tendency continued for 30 years in all the sub-sample periods, which is the longest among the six pairs. For the crude and its futures market pairs, its degree was assessed at the second highest, since their cointegration lasted for 26 years from 1993 to 2019 but became fragmented since 2019. However, the cointegration between WTI and Brent futures markets is persistent at the lowest extent, as these markets were in segregation for 16 years from 2003 to 2019 yet switched to be united since 2019. For the spot and futures cross pairs, its degree was evaluated at a moderate degree, since their cointegration became gradually weakened and finally disappeared. For the PT analysis results, we observed that for the recent half decade, the WTI spot dominates the Brent spot in the crude market, while Brent futures leads WTI futures in the derivatives market.

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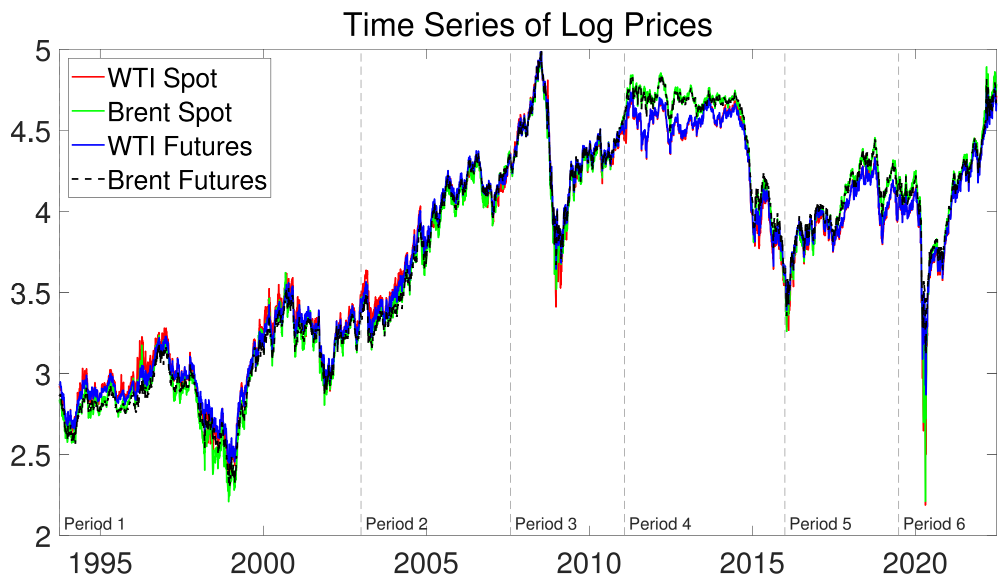

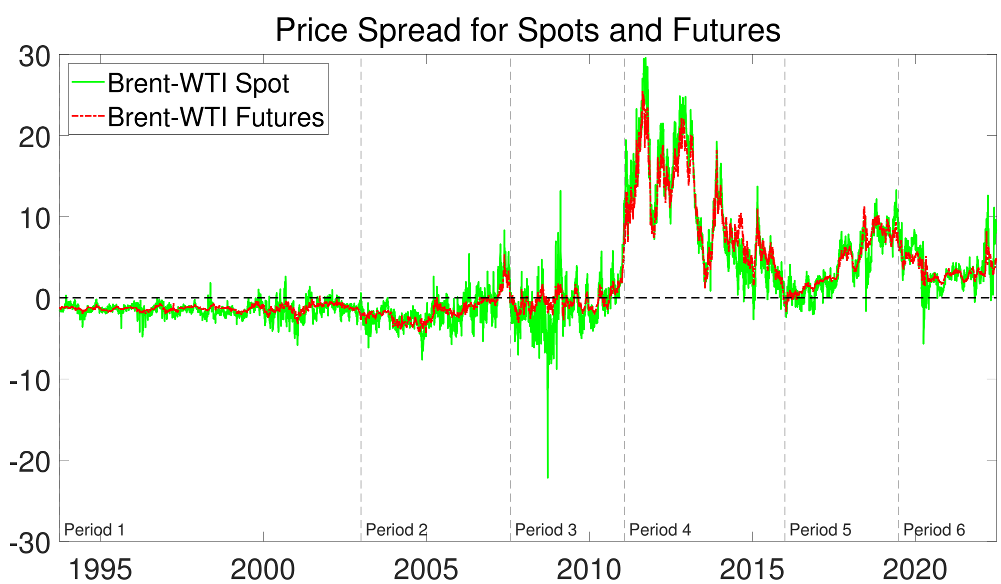

Figure 3: The daily log prices of WTI spot, Brent spot, WTI 3-month futures, and Brent 3-month futures (top), and the original price spreads of Brent-WTI spots and Brent-WTI futures (bottom) from 1 October 1993 to 27 June 2022 with displaying six sub-sample periods obtained by the Maki's structural break test.

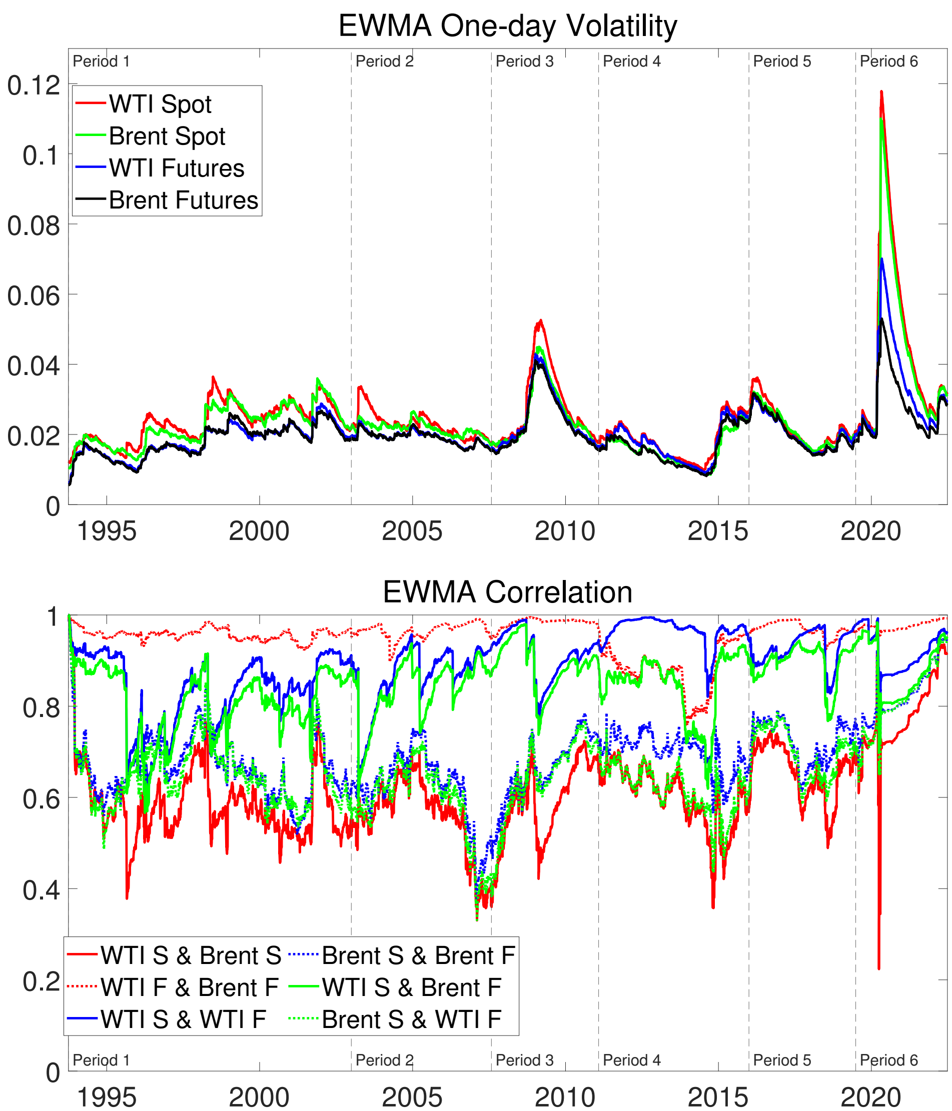

Figure 4: One-day volatility (top) and one-day correlation (bottom) estimated by the EWMA method with weighting rate 0.94 for WTI spot, Brent spot, WTI futures, and Brent futures series and the the six pairs of the time-series from 1993 to 2022.

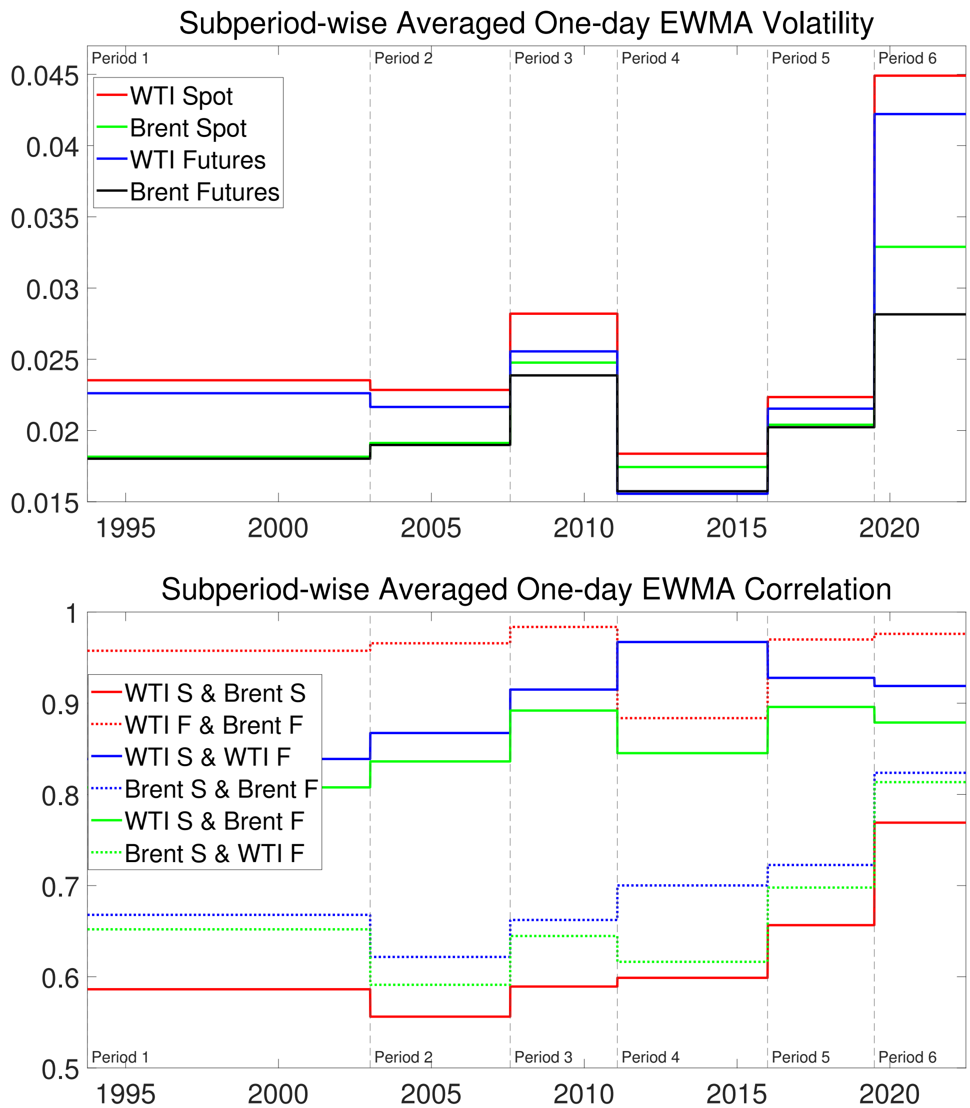

Figure 5: One-day volatility (top) and one-day correlation (bottom) estimated by the EWMA method with weighting rate 0.94 averaged over the six sub-sample periods for the four time-series (WTI spot, Brent spot, WTI futures, and Brent futures) and the six pairs of them from 1993 to 2022.

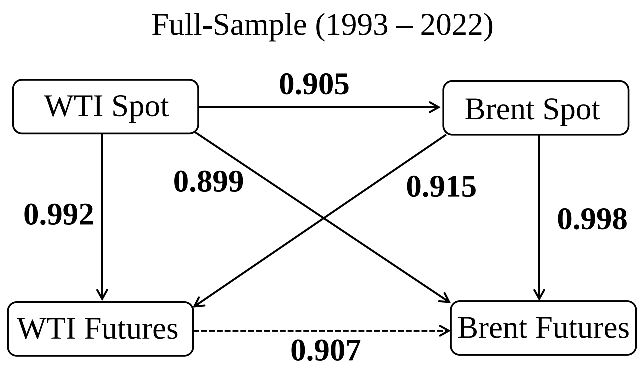

Figure 6: The full-sample result for the estimate of the cointegrating vector β and the hypothesis test for H^i_α, for $i = 1, 2$. Note that 'A $\rightarrow$ B' means B dominantly contribute to the long-run equilibrium, and the dotted line arrow represents the case that the dominant factor is not clearly determined as one of two prices.

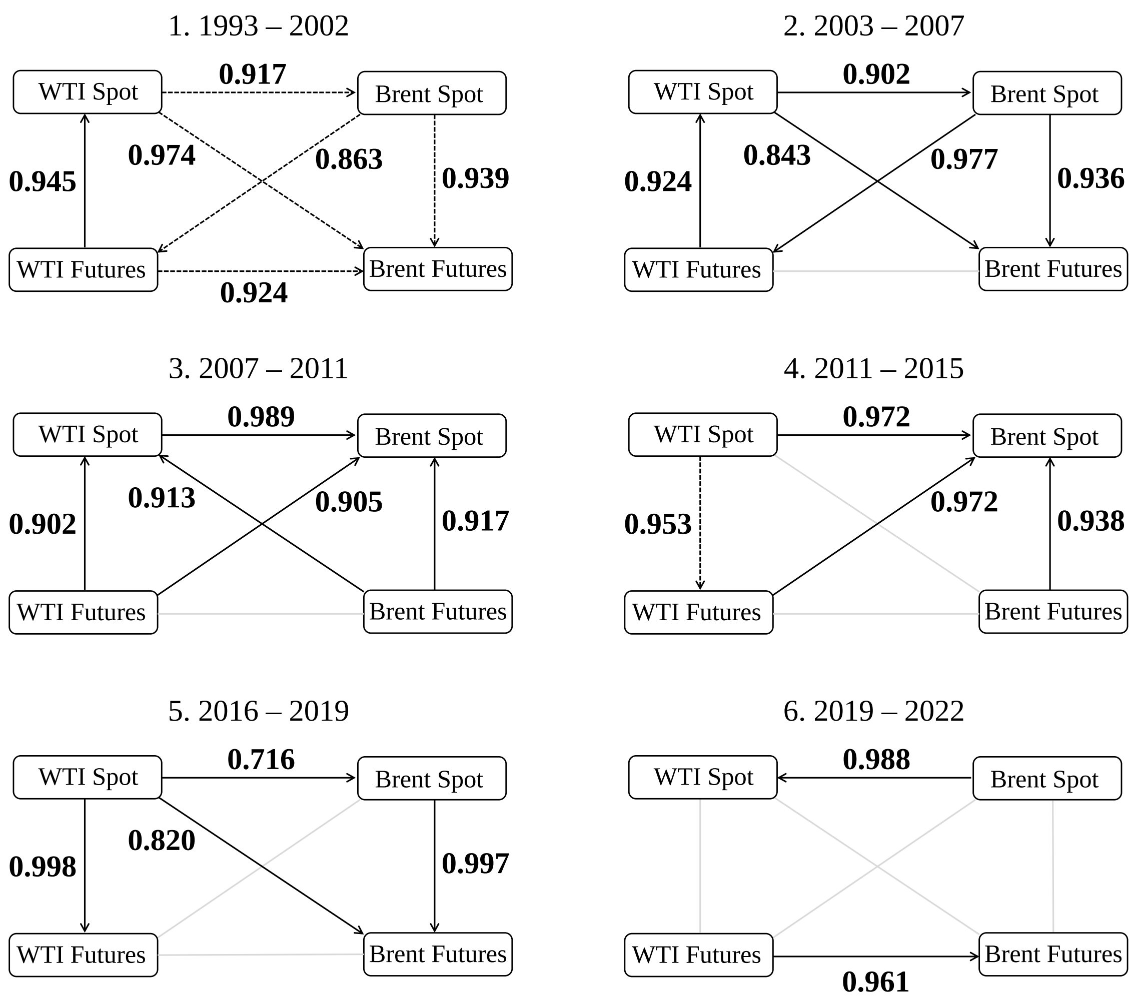

Figure 7: The sub-sample results for the estimates of $|\beta_2|$ and the hypothesis test of H_α^i, for $i = 1, 2$. Note that 'A $\rightarrow$ B' means B dominantly contribute to the long-run equilibrium, and the dotted line arrow represents the inconclusive case. The pale-gray line means no cointegration exists.