Modeling tail dependence using Gaussian copula

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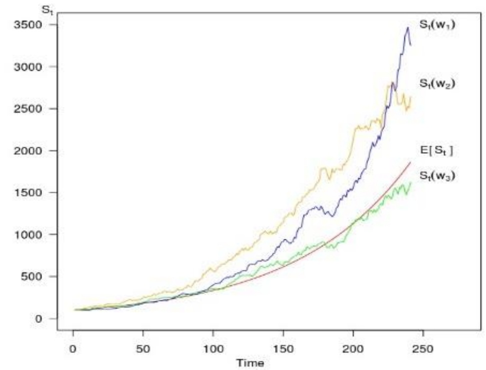
발표순서

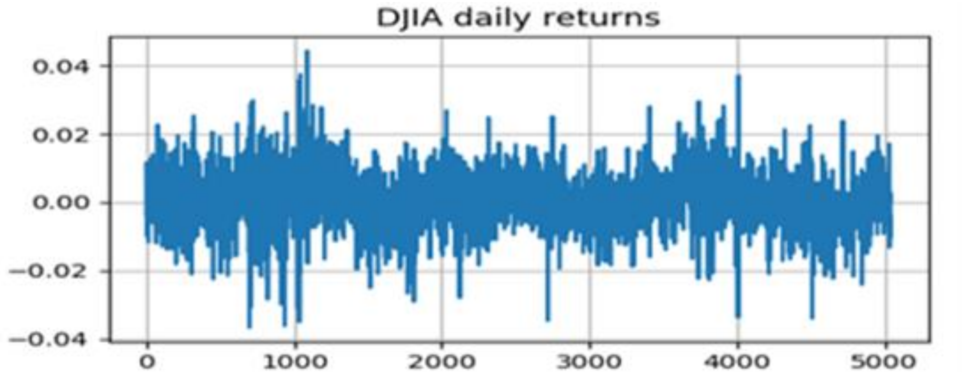
- 확률미분방정식
- 의존관계 모형화
- 일반화된 섭동가우시안 코퓰라
- 실증분석

확률미분방정식(I)

■ $$\frac{dS_t}{S_t} = \mu\,dt + \sigma\,dB_t \quad (\mu : \text{기대수익률},\ \sigma : \text{변동성})$$



■



$$\sigma \rightarrow \sigma(Y_t) \rightarrow \sigma(Y_t, Z_t)$$

확률미분방정식(II)

- $$\frac{dS_t}{S_t} = \mu dt + \sigma(Y_t, Z_t) dB_t$$

  $$dY_t = \alpha_1(m_1 - Y_t)dt + \sigma_1 dB_t^{(Y)} \qquad \sigma \rightarrow \sigma(Y_t) \rightarrow \sigma(Y_t, Z_t)$$

  $$dZ_t = \alpha_2(m_2 - Z_t)dt + \sigma_2 dB_t^{(Z)}$$

- Fast scale volatility factor

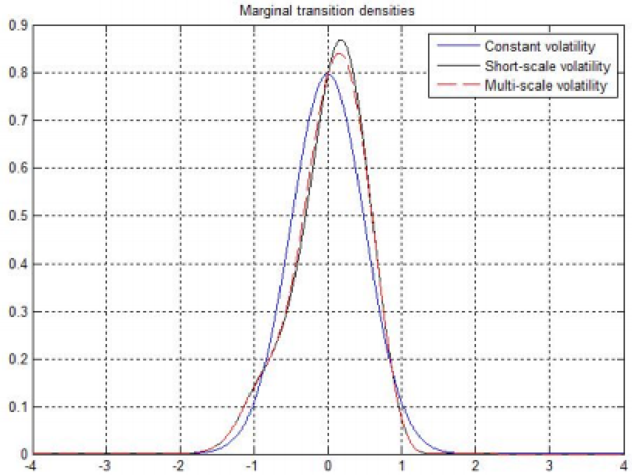
  : fast mean-reverting diffusion process

  Slow scale volatility factor

  : slowly varying diffusion process

확률미분방정식(III)

- Marginal distribution

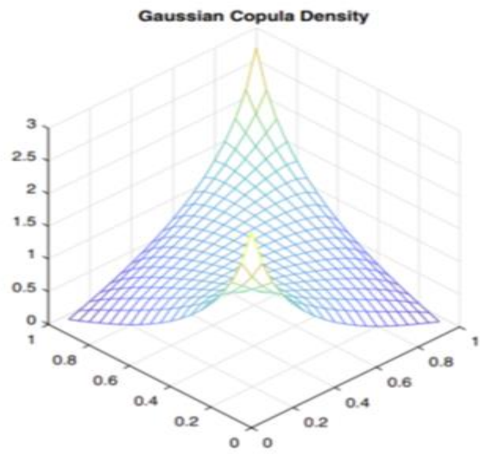


의존관계 모형화

- Linear correlation coefficient: : $-1 \le \rho(X, Y) \le 1$
  - The variances of X and Y be must finite
  - Independence of two random variables implies they are uncorrelated but zero correlation does not generally imply independence
  - Linear correlation is not invariant under non-linear strictly increasing transformations

- Copular function: Gaussian copula

$$dX_t^{(1)} = \sigma_1 dB_t^{(1)}$$
$$dX_t^{(2)} = \sigma_2 dB_t^{(2)}$$
$$dB_t^{(1)} dB_t^{(2)} = \rho\, dt$$



- Generally perturbed Gaussian copula

$$dX_t^{(1)} = f_1(Y_t, Z_t)dB_t^{(1)}$$
$$dX_t^{(2)} = f_2(Y_t, Z_t)dB_t^{(2)}$$
$$with \quad dY_t = \alpha_1(m_1 - Y_t)dt + \sigma_1 dB_t^{(Y)}, \quad dZ_t = \alpha_2(m_2 - Z_t)dt + \sigma_2 dB_t^{(Z)}$$

⇓

$$u^{\epsilon,\delta} := \mathbb{P}\{X_T^{(1)} \in d\xi_1, X_T^{(2)} \in d\xi_2 | \mathbf{X_t} = \mathbf{x}, \mathbf{Y_t} = \mathbf{y}, \mathbf{Z_t} = \mathbf{z}\}$$

- Generally perturbed Gaussian copula

$$dX_t^{(1)} = f_1(Y_t, Z_t)dB_t^{(1)}$$
$$dX_t^{(2)} = f_2(Y_t, Z_t)dB_t^{(2)}$$
$$\text{with} \quad dY_t = \alpha_1(m_1 - Y_t)dt + \sigma_1 dB_t^{(Y)}, \quad dZ_t = \alpha_2(m_2 - Z_t)dt + \sigma_2 dB_t^{(Z)}$$

⬇

$$dX_t^{(1)} = me^{\alpha_1 Y_t + \beta_1 Z_t} dB_t^{(1)}$$
$$dX_t^{(2)} = me^{\alpha_2 Y_t + \beta_2 Z_t} dB_t^{(2)}$$
$$\text{with} \quad dY_t = \frac{1}{\epsilon} Y_t \, dt + \frac{\nu_1 \sqrt{2}}{\sqrt{\epsilon}} dB_t^{(Y)}, \quad dZ_t = -\delta Z_t \, dt + \frac{\nu_2 \sqrt{2}}{\sqrt{\epsilon}} dB_t^{(Z)}$$

⬇

$$u^{\epsilon,\delta}(t, x_1, x_2, z) \approx u_{0,0}(t, x_1, x_2, z) + \sqrt{\epsilon} u_{1,0}(t, x_1, x_2, z) + \sqrt{\delta} u_{0,1}(t, x_1, x_2, z)$$

- The leading-order term

$$u_{0,0} = \frac{1}{2\pi\overline{\sigma}_1(z)\overline{\sigma}_2(z)(T-t)\sqrt{1-\overline{\rho}^2(z)}} \times \exp\left[-\frac{1}{2(1-\overline{\rho}^2(z))}\left\{\frac{(\xi_1-x_1)^2}{\overline{\sigma}_1^2(z)(T-t)} - 2\overline{\rho}(z)\frac{(\xi_1-x_1)(\xi_2-x_2)}{\overline{\sigma}_1(z)\overline{\sigma}_2(z)(T-t)} + \frac{(\xi_2-x_2)^2}{\overline{\sigma}_2^2(z)(T-t)}\right\}\right]$$

- The first order correction terms

$$\sqrt{\epsilon}u_{1,0} = -(T-t)\left(R_1(z)\frac{\partial^3}{\partial x_1^3} + R_2(z)\frac{\partial^3}{\partial x_2^3} + R_{12}(z)\frac{\partial^3}{\partial x_1 \partial x_2^2} + R_{21}(z)\frac{\partial^3}{\partial x_1^2 \partial x_2}\right)u_{0,0}$$

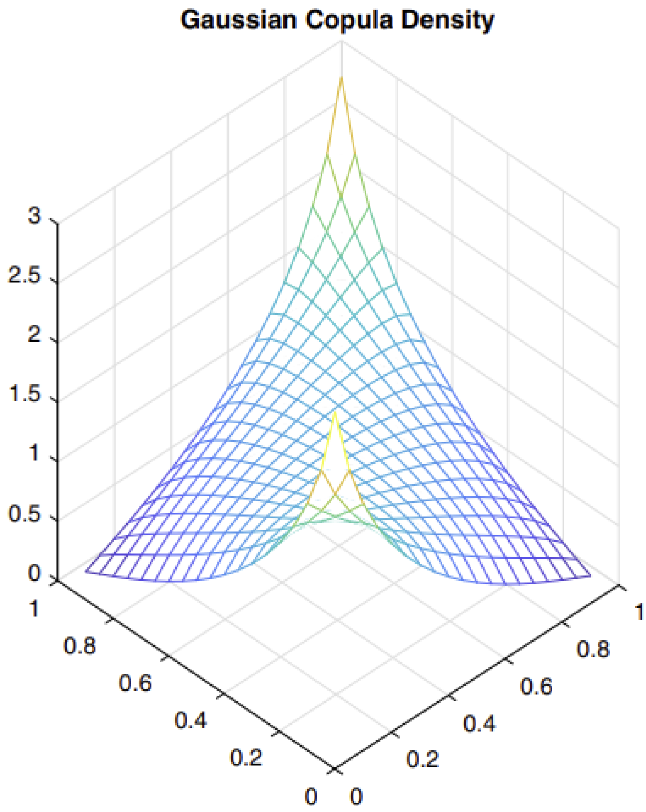
$$\sqrt{\delta}u_{0,1} = \frac{T-t}{2}\left(S_{12}(z)\frac{\partial^2}{\partial x_1 \partial z} + S_{21}(z)\frac{\partial^2}{\partial x_2 \partial z}\right)u_{0,0}$$

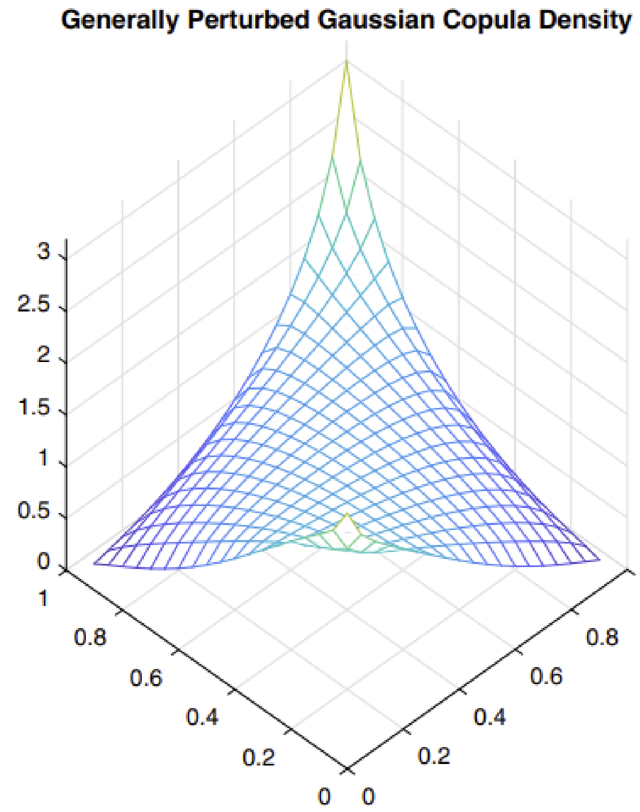
- Approximated transition density

$$\tilde{u}^{\epsilon,\delta} = \frac{1}{N}u_{0,0}\left(1 + \tanh\left(\sqrt{\epsilon}\frac{u_{1,0}}{u_{0,0}} + \sqrt{\delta}\frac{u_{0,1}}{u_{0,0}}\right)\right)$$

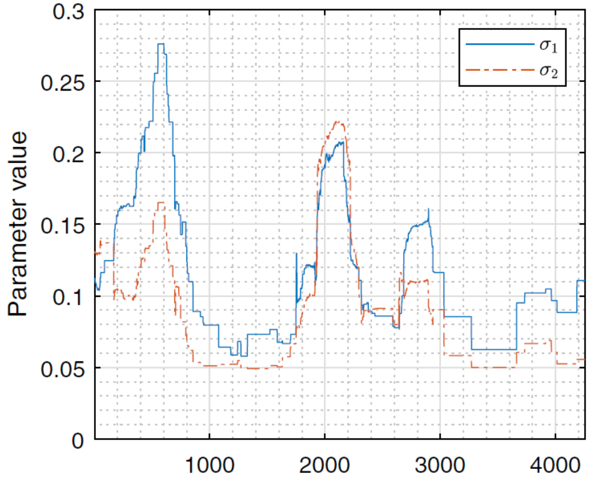
일반화된 섭동가우시안 코퓰라(IV)

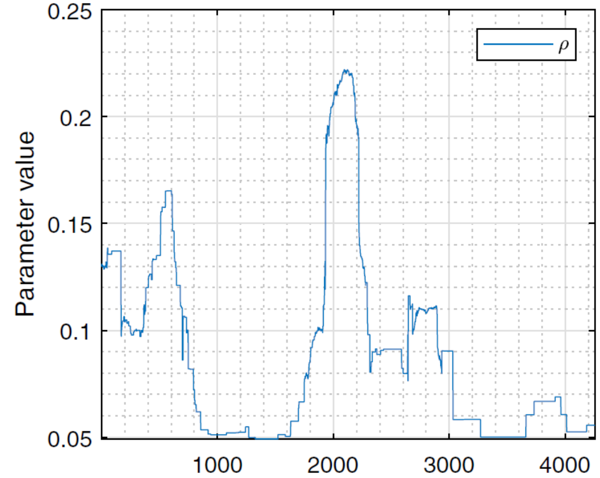
- 가우시안 코퓰라와 일반화된 섭동가우시안 코퓰라의 밀도함수 비교





- 2000년부터 2017년까지 SPX지수와 DAX지수 데이터를 통한 모형의 모수 추정값





질문 및 답변