Effectiveness of domain stabilization: Evidence from the KOSPI 200 options market

Geul Lee, Doojin Ryu*

Department of Economics, Sungkyunkwan University, Seoul, Korea

*Corresponding author. E-mail: doojin.ryu@gmail.com

ORCiD

Geul Lee (geul.lee.finance@gmail.com) https://orcid.org/0000-0002-3156-7806

Doojin Ryu (doojin.ryu@gmail.com): https://orcid.org/0000-0002-0059-4887

Highlights

·Domain stabilization mitigates the effect of truncation in the KOSPI 200 options market.

·Implied moments better explain underlying price levels and returns after stabilization.

·Domain stabilization improves underlying return predictability and forecasting performance.

Abstract

We evaluate the effectiveness of domain stabilization (DS) in mitigating the impact of truncation on model-free implied moment estimation in the KOSPI 200 index options market. Using a dataset spanning January 2015 to December 2023, we show that implied moments more effectively explain the contemporaneous underlying price level and log-returns after applying DS. Additionally, the in-sample return predictability and out-of-sample return forecasting performance of implied moment estimates improve with DS, though the improvement is somewhat weaker compared to that observed in the U.S. market. Our empirical results indicate that DS enhances the informational content of implied moment estimates, thereby improving their explanatory power for underlying prices and returns, as well as their predictive and forecasting performance.

Keywords: Deep-out-of-the-money options; Domain stabilization; Option-implied moments; S&P 500 options

JEL Classification: C14, C58, G13

1. Introduction

The option-implied risk-neutral density (RND) contains valuable information because it captures market participants' collective expectations about the probability distribution of the underlying asset's future price. As forward-looking instruments, options incorporate ex ante expectations that reflect a range of factors, including current market conditions, investor sentiment, and anticipated future volatility and tail risks. Despite its widespread application, a notable limitation of the RND lies in its representation as a continuous function, which complicates its direct use in empirical analysis. To overcome this challenge, researchers frequently employ the moments of the RND as discrete variables to capture the essential characteristics of the distribution. These moments offer a parsimonious and interpretable representation of the RND, enabling their application in a broad range of empirical studies investigating the informational content of option markets and their implications.

The development of the model-free implied moment estimators by Bakshi, Kapadia, and Madan (2003, BKM) has been a key driver behind the widespread use of the moments of the RND as variables in empirical research. These estimators facilitate the calculation of the volatility, skewness, and kurtosis of the RND without relying on restrictive assumptions about the underlying asset price dynamics, ensuring robustness against model misspecifications. By accurately capturing fundamental characteristics of the RND, the BKM estimators enable researchers to distill its key features into concise and interpretable variables that are well-suited for empirical analysis. Consequently, moments derived from BKM estimators have been extensively employed in studies examining market expectations, risk premia, and the pricing of higher-order risks, firmly establishing their role as indispensable tools in the investigation of option-implied information in financial markets.

The recent study of Lee, Ryu, and Yang (2024a, LRY) proposes a new truncation treatment method, termed domain stabilization (DS), to improve the consistency of BKM estimators and reduce variation in estimation errors. By stabilizing the degree of truncation over the sample period, DS seeks to reduce noise in estimation, thereby enhancing the informational value of implied moment estimates. LRY demonstrates that, when applied to S&P 500 index options data, DS significantly improves the in-sample predictive accuracy and out-of-sample forecasting performance of implied moment estimates for underlying log returns. Moreover, the test results of LRY indicate that implied moment estimates processed using DS outperform those obtained with alternative truncation treatment methods in both in-sample and out-of-sample tests. These findings highlight the potential of DS as a valuable tool for improving the information content of implied moment estimates and underscores the need for further investigation into its broader applications.

Motivated by these findings, this study extends LRY by evaluating the effectiveness of DS in KOSPI 200 index options market. Beyond assessing its performance in a new market, we expand the test methodology to evaluate the efficiency of DS from a broader range of perspectives. In addition to testing the in-sample and out-of-sample return predictive and forecasting performance, we examine the contemporaneous relationship between the underlying price and implied moment estimates to determine whether DS enhances the explanatory power of implied moment estimates on underlying price. To provide a detailed analysis, we investigate the contemporaneous relationship at both the levels and first-order differences, offering deeper insights into the effectiveness of DS.

The empirical findings of this study demonstrate that DS is an effective truncation treatment method for the KOSPI 200 index options market, consistent with the results reported by LRY for the S&P 500 index options market. Specifically, DS enhances the contemporaneous explanatory power of implied moment estimates for the underlying asset, both at the levels and first-order differences. Moreover, as shown by LRY, DS improves the in-sample return predictive accuracy and out-of-sample forecasting performance of implied moment estimates. While the enhancement generally becomes more pronounced as the intensity level of DS increases, it diminishes if DS is applied too intensively. The optimal improvement is achieved when the intensity level remains between 50 and 75 percent. The in-sample predictive and out-of-sample forecasting abilities of the implied moment estimates remain weaker in the KOSPI 200 index options market compared to the S&P 500 index options market, even after applying DS. This disparity may be attributed to differences in the degree of information asymmetry between the underlying and options markets in the two countries.

The remainder of this paper is organized as follows. Section 2 reviews the theoretical and empirical background, and Section 3 details the data sources, filtration criteria, and descriptive statistics for the KOSPI 200 options market dataset. Section 4 outlines the methodology for implied moment estimation, and Section 5 presents the empirical results. Section 6 concludes with a discussion of the findings and their implications.

2. Research background

BKM proposes a model-free implied moment estimation approach, defining the volatility, skewness, and kurtosis of the implied RND as functions of the fair values of volatility, cubic, and quartic contracts, denoted as V, W, anx X, respectively. These contracts are named based on their payoff functions, which correspond to R^2, R^3, and R^4, where R represents the underlying asset's holding period log return until maturity. At time t, BKM's implied moment estimators for maturity τ are defined as follows:

$$\mathrm{VOL}(t,\tau) \quad = [e^{r\tau}V(t,\tau) - \mu^2(t,\tau)]^{1/2}, \tag{1}$$

$$\mathrm{SKEW}(t,\tau) = \frac{e^{r\tau}W(t,\tau) - 3\mu(t,\tau)e^{r\tau}V(t,\tau) + 2\mu^3(t,\tau)}{[e^{r\tau}V(t,\tau) - \mu^2(t,\tau)]^{3/2}}, \tag{2}$$

$$\mathrm{KURT}(t,\tau) = \frac{e^{r\tau}X(t,\tau) - 4\mu(t,\tau)e^{r\tau}W(t,\tau) + 6e^{r\tau}\mu^2(t,\tau)V(t,\tau) - 3\mu^4(t,\tau)}{[e^{r\tau}V(t,\tau) - \mu^2(t,\tau)]^2}, \tag{3}$$

where r denotes the risk-free rate and $\mu(t,\tau)$ represents the underlying asset's expected holding period risk-neutral log return. BKM demonstrates that the fair values V, W, anx X can be derived from a continuum of OTM option prices as follows:

$$V(t,\tau) = \int_{S(t)}^{\infty} \frac{2\left(1 - ln\left[\frac{K}{S(t)}\right]\right)}{K^2} C(t,\tau;K)dK + \int_0^{S(t)} \frac{2\left(1 + ln\left[\frac{S(t)}{K}\right]\right)}{K^2} P(t,\tau;K)dK, \tag{4}$$

$$W(t,\tau) = \int_{S(t)}^{\infty} \frac{6\, ln\left[\frac{K}{S(t)}\right] - 3\left(ln\left[\frac{K}{S(t)}\right]\right)^2}{K^2} C(t,\tau;K)dK - \int_0^{S(t)} \frac{6\, ln\left[\frac{S(t)}{K}\right] + 3\left(ln\left[\frac{S(t)}{K}\right]\right)^2}{K^2} P(t,\tau;K)dK, \tag{5}$$

$$\begin{aligned} X(t,\tau) = &\int_{S(t)}^{\infty} \frac{12\left(ln\left[\frac{K}{S(t)}\right]\right)^2 - 4\left(ln\left[\frac{K}{S(t)}\right]\right)^3}{K^2} C(t,\tau;K)dK + \\ &\int_0^{S(t)} \frac{12\left(ln\left[\frac{S(t)}{K}\right]\right)^2 + 4\left(ln\left[\frac{S(t)}{K}\right]\right)^3}{K^2} P(t,\tau;K)dK, \end{aligned} \tag{6}$$

where $S(t)$ represents the underlying price at time t, and $C(t,\tau;K)$ and $P(t,\tau;K)$ denote the OTM call and put option prices for strike K, respectively. The fair value of $\mu(t,\tau)$ is approximated as:

$$\mu(t,\tau) = e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V(t,\tau) - \frac{e^{r\tau}}{6}W(t,\tau) - \frac{e^{r\tau}}{2}X(t,\tau). \tag{7}$$

When truncation occurs such that DOTM option prices are not observable, the observed OTM price data span a strike price domain of finite width, expressed as $[K_{\min}(t,\tau), K_{\max}(t,\tau)]$, where $K_{\min}(t,\tau)$ and $K_{\max}(t,\tau)$ represent the minimum and maximum strike prices of the integration domain, respectively. The existence of these finite and non-zero endpoints implies that, in estimating the fair values of $V(t,\tau)$, $W(t,\tau)$, and $X(t,\tau)$ as defined in Equations (4)–(6) under truncation, we are implicitly assuming that the OTM prices are zero for the strike price domains not covered by the observations. LRY demonstrates that this assumption is equivalent to truncating the implied RND itself, such that the risk-neutral probability is zero for the subset of strike prices outside the observed domain. Given this relationship, if implied moments are estimated over a period of time using daily option price observations with varying levels of truncation, the implied moments are effectively derived from daily implied RNDs that are truncated randomly. Therefore, proceeding with implied moment estimation under such truncation implies reliance on time-varying assumptions about the shape of implied RND. This variability undermines the consistency and suitability of the estimation approach, rendering it unsuitable for robust analysis. According to LRY, this issue can be addressed by ensuring consistency in the truncation assumption for the implied RND across the sample. This can be achieved by making the truncation on the integration domain consistent. LRY addresses this concern through a truncation treatment method called domain stabilization (DS). The detailed procedure for implementing DS is explained in Section 4, and its effectiveness is evaluated in the context of the KOSPI200 index options market in Section 5.

3. Data

This study uses daily closing price data from the KOSPI 200 index options market, covering the period from January 2015 to December 2023. The corresponding tick data, including transaction prices and the underlying index levels, are retrieved from the Korea Exchange. For each trading day, we identify the final transactions for each strike price that occur within the last minute before market close and use these observations exclusively for implied moment estimation. This approach has two key advantages: First, by eliminating the need to approximate option prices using bid and ask quotes, it avoids approximation errors inherent to such methods. Second, by considering prices only when the corresponding transactions occur during the last minute, it minimizes the risk of severe price staleness. We approximate the risk-free and dividend rates for each maturity by linearly interpolating the rates of adjacent maturities on the zero-coupon and futures-implied dividend curves. To approximate zero-coupon curves, we use the daily Korea Interbank Offered Rate (KORIBOR) data provided by the Bank of Korea. We collect daily closing prices of KOSPI 200 index futures from the Korea Exchange to estimate daily futures-implied continuously compounded dividend rate curves. After retrieving all the necessary datasets, we apply a series of data filtration criteria to ensure that the empirical analysis is conducted using only relevant and reliable option price observations, as outlined below. First, we remove options that are not OTM. Second, observations are discarded if the corresponding transaction price is below 0.03. Third, we exclude any observations with incomplete data entries. Finally, observations are discarded if they violate the no-arbitrage condition.

Table 1 presents the summary statistics for the final dataset, revealing three noteworthy findings. First, the magnitude of Black-Scholes d_1 is greater for OTM puts than calls, suggesting that OTM puts are deeper out-of-the-money on average. The asymmetry implies that the wider portion of the integration domain for implied moment estimation is covered by OTM puts, which is equivalent to a stronger impact of truncation on OTM calls. This tendency is also reflected in the percentile values. Second, the implied volatility level is higher for OTM puts than for OTM calls. This difference explains why truncation has a more severe impact on OTM call observations. Given the monotonic relationship between implied volatility and option price for a fixed level of moneyness, lower implied volatility translates into lower option prices, which increases the likelihood of exclusion by the minimum price filter. Third, compared to other observations, the observations with a time to maturity between 15 and 45 days do not exhibit significant differences. Therefore, although we focus on a single time to maturity of one month, it can be argued that the empirical results are representative of the entire KOSPI 200 index options market.

[Table 1 inserted about here]

Figure 1 illustrates the relationship between OTM option moneyness and the Black-Scholes implied volatility level across different sample subperiods. The figure reveals that, throughout the entire sample period, the implied volatility curve consistently exhibits a volatility smirk or skew, irrespective of the subperiod or the daily average implied volatility level. Based on prior literature that establishes a connection between implied moments and the shape of implied volatility curve (BKM; Zhang and Xiang, 2008), we conjecture that the implied RND is likely to be negatively skewed and leptokurtic, a hypothesis that we confirm in Section 5.

[Figure 1 inserted about here]

4. Methodology

The methodology applied to the empirical analysis in this study consists of three steps: constructing the implied volatility curve, estimating implied moments with or without applying DS, and conducting regression analysis to compare the information content of implied moment estimates with and without employing DS. To begin, we extract daily implied volatility curves for a one-month maturity from daily implied volatility surfaces constructed using observations from the final dataset. To construct the surfaces, we apply bilinear approximation to minimize the impact of abnormal observations and ensure feasibility even when data are available for no more than two maturities. The selection of a one-month maturity is motivated by two factors: the liquidity of markets for nearby maturities and the feasibility of interpolating between maturities shorter and longer than the target maturity. After extracting the implied volatility curve, we convert the implied volatility values into corresponding OTM option prices, further filtering out DOTM prices by reapplying a minimum price threshold of 0.03 to maintain consistency in data filtering. To minimize the effect of strike price discreteness, We set the strike price gap to 0.1, which is 1/25$^{\text{th}}$ of the original gap.

After constructing a series of daily implied volatility curves for a fixed maturity, we proceed with DS as follows. First, we measure the locations of the minimum and maximum endpoints of daily implied volatility curves with respect to moneyness, which correspond to the endpoints of integration domains for implied moment estimators, over the sample period. Following the approach of LRY, moneyness is expressed in terms of Black-Scholes d_1, defined as:

$$d_1[S(t), K, \sigma, r, \tau] = \frac{ln[S(t)/K] + [r + \sigma^2/2]\tau}{\sigma\sqrt{\tau}}, \tag{8}$$

where $S(t)$ denotes the underlying price, K representes the strike price, σ is the volatility of the underlying returns, r represents the risk-free rate, and τ denotes the time to maturity. Next, we determine the percentiles of the minimum and maximum endpoints, considering the extent to which OTM price observations will be further discarded. Once the percentiles are chosen, we apply DS by discarding observations whose strike prices fall outside these percentile values. Specifically, for an intensity level of i percent, we discard an OTM put price observation if the corresponding d_1 exceeds the i^{th} percentile of daily maximum endpoint observations. Similarly, we exclude an OTM call price observation if the corresponding d_1 is more negative than the $(100 - i)^{\text{th}}$

percentile of daily minimum endpoint observations. If there are insufficient observations so that the daily implied volatility curve do not cover the entire range between the $(100-i)^{\text{th}}$ percentile of daily minimum endpoint observations and the i^{th} percentile of daily maximum endpoint observations, we employ flat extrapolation to extend the implied volatility curve up to the two percentile values. After treatments, we obtain a series of implied volatility curve with consistent endpoints across the sample period.

Finally, we estimate implied volatility, skewness, and kurtosis using the BKM estimators with and without DS applied, and conduct a set of regression analyses to investigate whether DS enhances the information content of implied moment estimates. We begin the regression analysis by examining the explanatory power of implied moment levels for the underlying price level using the following model:

$$ln\,[S(t)] = \alpha + \beta_0 \cdot VOL(t) + \beta_1 \cdot SKEW(t) + \beta_2 \cdot KURT(t) + \varepsilon(t), \quad (9)$$

where $\ln[S(t)]$ is the natural logarithm of the underlying KOSPI 200 index level on day t, $VOL(t)$, $SKEW(t)$, and $KURT(t)$ denote the levels of implied volatility, skewness, and kurtosis estimates on day t, respectively. Next, we investigate the explanatory power of the first-order differences of implied moments for the underlying log-returns with the following model:

$$\Delta ln\,[S(t)] = \alpha + \beta_0 \cdot \Delta VOL(t) + \beta_1 \cdot \Delta SKEW(t) + \beta_2 \cdot \Delta KURT(t) + \varepsilon(t), \quad (10)$$

where Δ is the first-order difference operator. Finally, we conduct in-sample return predictability and out-of-sample return forecasting ability tests using the following model:

$$\begin{aligned}\Delta ln\,[S(t)] = \alpha + \beta_0 \cdot \Delta ln\,[S(t-1)] + \gamma_0 \cdot \Delta VOL(t-1) + \gamma_1 \cdot \Delta SKEW(t-1) + \gamma_2 \cdot \\ \Delta KURT(t) + \varepsilon(t).\end{aligned} \quad (11)$$

5. Empirical results

5.1. Degree of truncation and moment estimates

Table 2 presents the summary statistics of implied moment estimates, with DS not applied during their collection. The table reveals that the implied RND is generally negatively skewed and leptokurtic, consistent with findings from several previous studies. Notably, the log-return distribution of the underlying index also exhibits negative skewness and leptokurtosis, which can be regarded as a primary determinant of the moments of the implied RND. However, it is noteworthy that the kurosis of the underlying log-return distribution is significantly higher than the implied kurtosis on average, a difference that may be attributed to truncation.

[Table 2 inserted about here]

Figure 2 illustrates the dynamics of daily implied moment estimates over the sample period. Panel A presents the implied volatility estimate dynamics, which demonstrates a time trend without significant noise. The clear dynamics suggest that the implied volatility estimates are not significantly affected by noise factors such as truncation, which is consistent with LRY. By comparison, Panels B and C, which depict the implied skewness and kurtosis estimate dynamics, respectively, reveal that the estimate dynamics are heavily affected by noisy fluctuations. These fluctuations are likely attributable to variations in the degree of truncation, as also noted in LRY.

[Figure 2 inserted about here]

Figure 3 depicts the changes in the shape of the integration domain, measured in various ways, over the sample period. Panels A and B show that when we the width of integration domain is measured in nominal price terms, it begins to expand notably from 2020, coinciding with the onset of the COVID-19 period. This widening is primarily attributed to an increase in implied volatility, as evidenced in Panel A of Figure 2. In contrast, Panel C of Figure 3, which examines the integration domain in terms of Black-Scholes d_1, the noisy fluctuations are more pronounced, indicating that these fluctuations are a key factor driving the variability in the integration domain. This, in turn, may contribute to the noisy dynamics observed in the implied higher moment estimates, as noted by LRY.

[Figure 3 inserted about here]

6. Conclusion

This study examines the whether DS effectively enhances the informational content of model-free implied moment estimates in the KOSPI 200 index options market. In addition to evaluating DS in a new market context, we conduct the evaluation from a wider range of perspectives. Specifically, we assess not only the in-sample and out-of-sample predictive and forecasting abilities but also the contemporaneous explanatory power of implied moment estimates and their first-order differences with respect to the underlying log-price and log-returns. The empirical results demonstrate that DS improves the contemporaneous explanatory power, enhances in-sample predictive accuracy, and strengthens out-of-sample forecasting performance of implied moment estimates. These findings suggest that DS is an effective truncation treatment method, aligning with the results of LRY. Future research could explore whether the return predictive and forecasting abilities of implied moment estimates improve with DS even on an intra-day basis.

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Table 1. Sample summary statistics

Panel A. Black-Scholes d_1

	Maturity between 15 and 45 days		Other maturities	
	Puts	Calls	Puts	Calls
Mean	1.376	-1.130	1.405	-1.178
Median	1.409	-1.125	1.425	-1.187
Std. dev.	0.676	0.671	0.621	0.611
5th pct.	0.241	-2.221	0.323	-2.160
95th pct.	2.443	-0.075	2.379	-0.141
Skewness	-0.065	-0.064	-0.110	0.032
Kurtosis	2.100	1.967	2.328	2.230
# of obs.	32,564	21,775	21,461	16,075

Panel B. Black-Scholes implied volatility

	Maturity between 15 and 45 days		Other maturities	
	Puts	Calls	Puts	Calls
Mean	0.229	0.153	0.233	0.155
Median	0.202	0.138	0.200	0.137
Std. dev.	0.105	0.064	0.119	0.065
5th pct.	0.122	0.098	0.126	0.099
95th pct.	0.435	0.253	0.465	0.275
Skewness	2.129	3.835	2.621	2.889
Kurtosis	9.555	24.441	12.772	14.030
# of obs.	32,564	21,775	21,461	16,075

Notes: This table presents summary statistics for Black-Scholes d_1 and implied volatility of the KOSPI 200 index options dataset used in this study. After data filtration, the daily sample of option prices comprises 91,875 observations, spanning from January 2015 to December 2023. The following criteria are applied to exclude inappropriate observations: (1) the option is not out-of-the-money; (2) the closing price is below 0.03; (3) any corresponding data entries are missing; and (4) the no-arbitrage condition is violated.

Table 2. Underlying price and implied moment estimates

	Panel A. Levels				Panel B. Daily first-order differences			
	ln(*S*)	*VOL*	*SKEW*	*KURT*	$\Delta ln(S) \cdot 10^2$	ΔVOL	$\Delta SKEW$	$\Delta KURT$
Mean	5.694	0.164	-0.988	5.081	0.022	0.000	0.000	0.000
Median	5.685	0.146	-0.943	4.665	0.059	0.000	0.005	-0.033
Std. dev.	0.159	0.062	0.414	1.857	1.209	0.016	0.245	1.350
5th pct.	5.476	0.109	-1.740	3.127	-1.831	-0.021	-0.398	-2.044
95th pct.	6.021	0.272	-0.405	8.550	1.809	0.023	0.394	2.144
Skewness	0.505	3.109	-0.843	2.130	-0.729	1.944	-0.201	0.221
Kurtosis	2.642	17.918	4.434	10.807	11.792	33.729	4.418	6.222
# of obs.	1,703	1,703	1,703	1,703	1,702	1,702	1,702	1,702

Notes: This table presents summary statistics for the daily closing underlying log-prices and implied moment estimates, calculated without applying domain stabilization. To estimate the implied moments, we approximate option prices for a one-month time to maturity based on daily implied volatility surfaces. Panels A and B report the summary statistics for the levels and first-order differences, respectively. In Panel B, only cases with exactly a one-trading-day gap between two sample days are considered.

Table 3. Moment estimates after domain stabilization

Panel A. Volatility

	Level					First-order difference				
Intensity	0%	25%	50%	75%	100%	0%	25%	50%	75%	100%
Mean	0.167	0.166	0.164	0.162	0.130	0.000	0.000	0.000	0.000	0.000
Median	0.148	0.147	0.146	0.145	0.117	0.000	0.000	0.000	0.000	0.000
Std. dev.	0.065	0.065	0.064	0.063	0.048	0.018	0.018	0.017	0.017	0.013
5th pct.	0.111	0.110	0.109	0.107	0.087	-0.022	-0.022	-0.021	-0.021	-0.016
95th pct.	0.275	0.271	0.267	0.263	0.209	0.024	0.023	0.023	0.022	0.017
Skewness	3.518	3.552	3.576	3.592	3.381	3.580	3.563	3.582	3.625	3.649
Kurtosis	22.447	22.949	23.314	23.628	21.847	68.870	70.442	71.998	73.769	71.450
# of obs.	1,703	1,703	1,703	1,703	1,703	1,702	1,702	1,702	1,702	1,702

Panel B. Skewness

	Level					First-order difference				
Intensity	0%	25%	50%	75%	100%	0%	25%	50%	75%	100%
Mean	-1.072	-1.018	-0.958	-0.879	-0.307	0.000	0.000	0.000	0.000	0.000
Median	-1.018	-0.982	-0.927	-0.852	-0.314	0.004	0.005	0.006	0.004	-0.001
Std. dev.	0.430	0.337	0.288	0.240	0.073	0.217	0.161	0.134	0.110	0.041
5th pct.	-1.898	-1.658	-1.504	-1.326	-0.411	-0.354	-0.263	-0.219	-0.180	-0.055
95th pct.	-0.484	-0.532	-0.536	-0.516	-0.180	0.324	0.245	0.203	0.165	0.053
Skewness	-0.978	-0.643	-0.600	-0.584	1.009	-0.275	-0.296	-0.204	-0.051	2.285
Kurtosis	4.571	3.432	3.387	3.435	6.319	5.582	5.157	5.537	6.148	24.277
# of obs.	1,703	1,703	1,703	1,703	1,703	1,702	1,702	1,702	1,702	1,702

Panel C. Kurtosis

	Level					First-order difference				
Intensity	0%	25%	50%	75%	100%	0%	25%	50%	75%	100%
Mean	6.089	5.289	4.710	4.043	1.095	0.000	0.000	0.000	0.000	0.000
Median	5.599	5.107	4.602	3.954	1.092	-0.037	-0.008	-0.003	0.000	0.000
Std. dev.	2.053	1.115	0.792	0.547	0.072	1.273	0.635	0.425	0.276	0.045
5th pct.	4.027	3.867	3.677	3.333	0.993	-1.898	-1.036	-0.701	-0.447	-0.071
95th pct.	9.979	7.571	6.340	5.174	1.223	1.971	1.034	0.712	0.446	0.070
Skewness	2.242	1.124	1.057	1.086	0.102	0.101	0.197	0.153	0.054	0.000
Kurtosis	11.021	4.732	4.576	4.765	6.692	7.766	4.201	4.486	5.104	6.810
# of obs.	1,703	1,703	1,703	1,703	1,703	1,702	1,702	1,702	1,702	1,702

Notes: This table provides summary statistics for the implied volatility, skewness, and kurtosis estimates after applying domain stabilization at different intensity levels. 0%, 25%, 50%, 75%, and 100% indicate the intensity levels.

Table 4. Correlations among underlying log-prices and implied moment estimates

Panel A. No stabilization

	Level					First-order difference			
	ln(*S*)	*VOL*	*SKEW*	*KURT*		Δ*ln*(*S*)	Δ*VOL*	Δ*SKEW*	Δ*KURT*
ln(*S*)	1.000	-	-	-	Δ*ln*(*S*)	1.000	-	-	-
VOL	0.055	1.000	-	-	Δ*VOL*	-0.633	1.000	-	-
SKEW	0.041	-0.330	1.000	-	Δ*SKEW*	0.096	-0.251	1.000	-
KURT	0.034	0.175	-0.860	1.000	Δ*KURT*	0.003	0.138	-0.865	1.000

Panel B. 0% stabilization

	Level					First-order difference			
	ln(*S*)	*VOL*	*SKEW*	*KURT*		Δ*ln*(*S*)	Δ*VOL*	Δ*SKEW*	Δ*KURT*
ln(*S*)	1.000	-	-	-	Δ*ln*(*S*)	1.000	-	-	-
VOL	0.041	1.000	-	-	Δ*VOL*	-0.639	1.000	-	-
SKEW	0.075	-0.386	1.000	-	Δ*SKEW*	0.204	-0.287	1.000	-
KURT	-0.009	0.204	-0.877	1.000	Δ*KURT*	-0.043	0.124	-0.877	1.000

Panel C. 25% stabilization

	Level					First-order difference			
	ln(*S*)	*VOL*	*SKEW*	*KURT*		Δ*ln*(*S*)	Δ*VOL*	Δ*SKEW*	Δ*KURT*
ln(*S*)	1.000	-	-	-	Δ*ln*(*S*)	1.000	-	-	-
VOL	0.041	1.000	-	-	Δ*VOL*	-0.642	1.000	-	-
SKEW	0.092	-0.404	1.000	-	Δ*SKEW*	0.298	-0.343	1.000	-
KURT	-0.017	0.241	-0.885	1.000	Δ*KURT*	-0.149	0.177	-0.847	1.000

Panel D. 50% stabilization

	Level					First-order difference			
	ln(*S*)	*VOL*	*SKEW*	*KURT*		Δ*ln*(*S*)	Δ*VOL*	Δ*SKEW*	Δ*KURT*
ln(*S*)	1.000	-	-	-	Δ*ln*(*S*)	1.000	-	-	-
VOL	0.041	1.000	-	-	Δ*VOL*	-0.642	1.000	-	-
SKEW	0.108	-0.409	1.000	-	Δ*SKEW*	0.298	-0.343	1.000	-
KURT	-0.030	0.250	-0.874	1.000	Δ*KURT*	-0.149	0.177	-0.847	1.000

Panel E. 75% stabilization

	Level					First-order difference			
	ln(*S*)	*VOL*	*SKEW*	*KURT*		Δ*ln*(*S*)	Δ*VOL*	Δ*SKEW*	Δ*KURT*
ln(*S*)	1.000	-	-	-	Δ*ln*(*S*)	1.000	-	-	-
VOL	0.041	1.000	-	-	Δ*VOL*	-0.642	1.000	-	-
SKEW	0.124	-0.405	1.000	-	Δ*SKEW*	0.315	-0.350	1.000	-
KURT	-0.042	0.252	-0.857	1.000	Δ*KURT*	-0.171	0.178	-0.818	1.000

Panel F. 100% stabilization

	Level					First-order difference			
	ln(*S*)	*VOL*	*SKEW*	*KURT*		Δ*ln*(*S*)	Δ*VOL*	Δ*SKEW*	Δ*KURT*
ln(*S*)	1.000	-	-	-	Δ*ln*(*S*)	1.000	-	-	-
VOL	0.052	1.000	-	-	Δ*VOL*	-0.645	1.000	-	-
SKEW	0.286	0.151	1.000	-	Δ*SKEW*	0.112	-0.076	1.000	-
KURT	-0.050	0.024	-0.596	1.000	Δ*KURT*	-0.082	0.008	-0.572	1.000

Notes: This table presents the estimated correlation among the log price, implied volatility, implied skewness, and implied kurtosis, which are denoted as *ln*(*S*), *VOL*, *SKEW*, and *KURT*, respectively.

Table 5. Explanatory power of implied moments for underlying log-price: Levels

		No stabilization	0 percent stabilization	25 percent stabilization	50 percent stabilization	75 percent stabilization	100 percent stabilization
VOL(*t*)		0.294	0.324	0.360	0.361	0.360	-0.042
		(0.91)	(1.03)	(1.17)	(1.17)	(1.16)	(-0.55)
SKEW(*t*)		0.134***	0.151***	0.229***	0.255***	0.287***	0.869***
		(2.63)	(2.80)	(3.33)	(3.40)	(3.49)	(13.71)
KURT(*t*)		0.027***	0.025***	0.054***	0.068***	0.085***	0.422***
		(2.87)	(2.61)	(2.98)	(2.89)	(2.76)	(6.56)
Intercept		5.641***	5.650***	5.583***	5.559***	5.542***	5.505***
		(125.27)	(122.01)	(94.09)	(80.61)	(68.92)	(90.89)
# of obs.		1,703	1,703	1,703	1,703	1,703	1,703
Unadjusted R^2		0.031	0.033	0.044	0.045	0.047	0.104
AIC	Value	-1470.25	-1474.30	-1494.15	-1495.99	-1499.02	-1604.64
	Diff.	0.00	-4.05	-23.90	-25.74	-28.77	-134.39
BG		1671.1***	1678.6***	1674.1***	1674.6***	1674.6***	1636.9***
BP		25.3***	9.9***	8.9***	7.3***	5.5**	43.5***

Notes: This table presents the regression results of the underlying log-price on the levels of implied moments, using domain stabilization at various intensity levels. The dependent variable, $\ln[S(t)]$, represents the natural logarithm of the S&P 500 index on day t. $VOL(t)$, $SKEW(t)$, and $KURT(t)$ denote the daily levels of implied volatility, skewness, and kurtosis estimates on day t, respectively. Newey-West standard errors are employed to address residual autocorrelation and heteroskedasticity, with a lag length of nine selected according to Andrew's rule. The difference in information criteria (Diff.) reflects the variation in OLS-based Akaike information criterion (AIC) values relative to the no stabilization case. The Breusch-Godfrey (BG) and Breusch-Pagan (BP) tests report $\chi^2(1)$ statistics for testing autocorrelation and heteroscadasticity, respectively. t-statistics are presented in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 6. Explanatory power of implied moments for underlying log-price: First-order differences

		No stabilization	0 percent stabilization	25 percent stabilization	50 percent stabilization	75 percent stabilization	100 percent stabilization
ΔVOL(*t*)		-47.23***	-40.84***	-40.99***	-41.47***	-42.34***	-59.99***
		(-19.20)	(-16.88)	(-16.39)	(-16.35)	(-16.34)	(-19.06)
ΔSKEW(*t*)		0.271	1.452***	1.828***	1.838***	1.761***	0.856
		(0.90)	(3.96)	(3.52)	(3.13)	(2.65)	(0.70)
ΔKURT(*t*)		0.124**	0.247***	0.370***	0.368**	0.288	-1.621**
		(2.54)	(4.31)	(3.28)	(2.37)	(1.31)	(-2.11)
Intercept		0.024	0.024	0.024	0.024	0.023	0.024
		(0.99)	(0.99)	(0.99)	(0.99)	(1.00)	(1.00)
# of obs.		1,702	1,702	1,702	1,702	1,702	1,702
Unadjusted R^2		0.410	0.421	0.423	0.423	0.423	0.423
AIC	Value	4585.56	4549.60	4546.98	4547.30	4547.05	4548.17
	Diff.	0.00	-35.96	-38.58	-38.26	-38.51	-37.39
BG		6.4**	6.7***	6.1**	6.2**	6.4**	6.5**
BP		1.1	75.3***	70.1***	70.9***	71.6***	91.7***

Notes: This table presents the regression results of the underlying log-return on the first-order differences of implied moments, using domain stabilization at various intensity levels. The dependent variable, 100·$\Delta lnS(t)$, represents log-return of the S&P 500 index on day t expressed in percentage. $\Delta VOL(t)$, $\Delta SKEW(t)$, and $\Delta KURT(t)$ denote the daily first-order differences of implied volatility, skewness, and kurtosis estimates on day t, respectively. Newey-West standard errors are employed to address residual autocorrelation and heteroskedasticity, with a lag length of nine selected according to Andrew's rule. The difference in information criteria (Diff.) reflects the variation in OLS-based Akaike information criterion (AIC) values relative to the no stabilization case. The Breusch-Godfrey (BG) and Breusch-Pagan (BP) tests report $\chi^2(1)$ statistics for testing autocorrelation and heteroscadasticity, respectively. t-statistics are presented in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 7. In-sample return predictive ability of implied moments

		No stabilization	0 percent stabilization	25 percent stabilization	50 percent stabilization	75 percent stabilization	100 percent stabilization
ΔlnS(t-1)		0.021	0.007	0.003	0.003	0.003	-0.002
		(0.61)	(0.23)	(0.10)	(0.11)	(0.10)	(-0.05)
ΔVOL(*t*-1)		-1.411	-3.341	-2.961	-2.904	-2.927	-3.763
		(-0.36)	(-0.76)	(-0.65)	(-0.64)	(-0.65)	(-0.68)
ΔSKEW(*t*-1)		-0.738*	-0.612	-0.153	0.037	0.163	-0.716
		(-1.76)	(-1.51)	(-0.33)	(0.08)	(0.33)	(-0.89)
ΔKURT(*t*-1)		-0.092	-0.067	0.103	0.239*	0.395*	0.957
		(-1.49)	(-1.18)	(1.12)	(1.84)	(1.86)	(0.95)
Intercept		0.022	0.022	0.022	0.022	0.022	0.023
		(0.74)	(0.76)	(0.76)	(0.76)	(0.76)	(0.76)
# of obs.		1,701	1,701	1,701	1,701	1,701	1,701
Unadjusted R^2		0.004	0.002	0.004	0.005	0.005	0.002
AIC	Value	5472.94	5476.94	5473.43	5471.21	5471.39	5476.32
	Diff.	0.00	4.00	0.49	-1.73	-1.55	3.38
BG		0.7	7.7***	3.8*	3.5*	3.3*	13.1***
BP		37.5***	60.0***	52.3***	61.0***	75.8***	70.8***

Notes: This table presents the regression results of the underlying log-return on the first-order differences of implied moments, using domain stabilization at various intensity levels. The dependent variable, $\Delta lnS(t)$, represents the log-return of the S&P 500 index on day t. The lagged log-return, $\Delta lnS(t-1)$, is included as an independent variable to control for return reversals. $\Delta VOL(t-1)$, $\Delta SKEW(t-1)$, and $\Delta KURT(t-1)$ denote the lagged daily first-order differences of implied volatility, skewness, and kurtosis estimates, respectively. Newey-West standard errors are employed to address residual autocorrelation and heteroskedasticity, with a lag length of nine selected according to Andrew's rule. The difference in information criteria (Diff.) reflects the variation in OLS-based Akaike information criterion (AIC) values relative to the no stabilization case. The Breusch-Godfrey (BG) and Breusch-Pagan (BP) tests report $\chi^2(1)$ statistics for testing autocorrelation and heteroscadasticity, respectively. t-statistics are presented in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 8. Out-of-sample return forecasting ability of implied moments after stabilization

Rolling window length	R^2_{OS}					
	No stabilization	0 percent stabilization	25 percent stabilization	50 percent stabilization	75 percent stabilization	100 percent stabilization
5 months	-9.79	-13.96	-13.40	-13.12	-13.51	-13.06
10 months	-4.77	-7.66	-7.86	-7.76	-7.96	-6.58
15 months	-3.97	-6.07	-5.94	-5.75	-5.87	-4.69
20 months	-2.36	-4.21	-4.12	-4.01	-4.11	-3.17
25 months	-1.93	-3.40	-3.09	-2.98	-3.08	-2.65
30 months	-1.98	-2.80	-2.37	-2.23	-2.33	-1.95
35 months	-1.64	-2.12	-1.74	-1.61	-1.74	-1.62
40 months	-1.53	-1.91	-1.62	-1.52	-1.66	-1.51
45 months	-1.27	-1.59	-1.33	-1.20	-1.26	-1.27
50 months	-0.93	-1.29	-1.05	-0.88	-0.91	-1.05
55 months	-1.01	-1.35	-1.10	-0.87	-0.86	-1.03
60 months	-0.86	-1.00	-0.53	-0.36	-0.40	-0.71
65 months	-1.33	-1.16	-0.45	-0.25	-0.31	-0.57
70 months	-1.00	-0.66	-0.28	-0.21	-0.25	-0.38
75 months	-1.15	-0.56	-0.21	0.11	0.05	-0.53
80 months	-2.29	-0.84	0.07	0.45	0.59	-0.14

Notes: This table presents the results of the out-of-sample return forecasting ability test. Following Campbell and Thompson (2008), we report the R^2_{OS} statistic, which is defined as $R^2_{OS} = 1 - [\sum_{t=1}^{T}(r_t - \hat{r}_t)^2]/[\sum_{t=1}^{T}(r_t - \overline{r}_t)^2]$, where $\hat{r}_t$ is the fitted value derived from a predictive regression estimated through the rolling window that ends at time $t-1$, and $\overline{r}_t$ is the benchmark value for the rolling window. Benchmark value is defined as the historical mean log-return. A positive value of R^2_{OS} indicates that the predictive regression produces a lower mean squared prediction error than the benchmark value. The value of R^2_{OS} is expressed as a percentage.

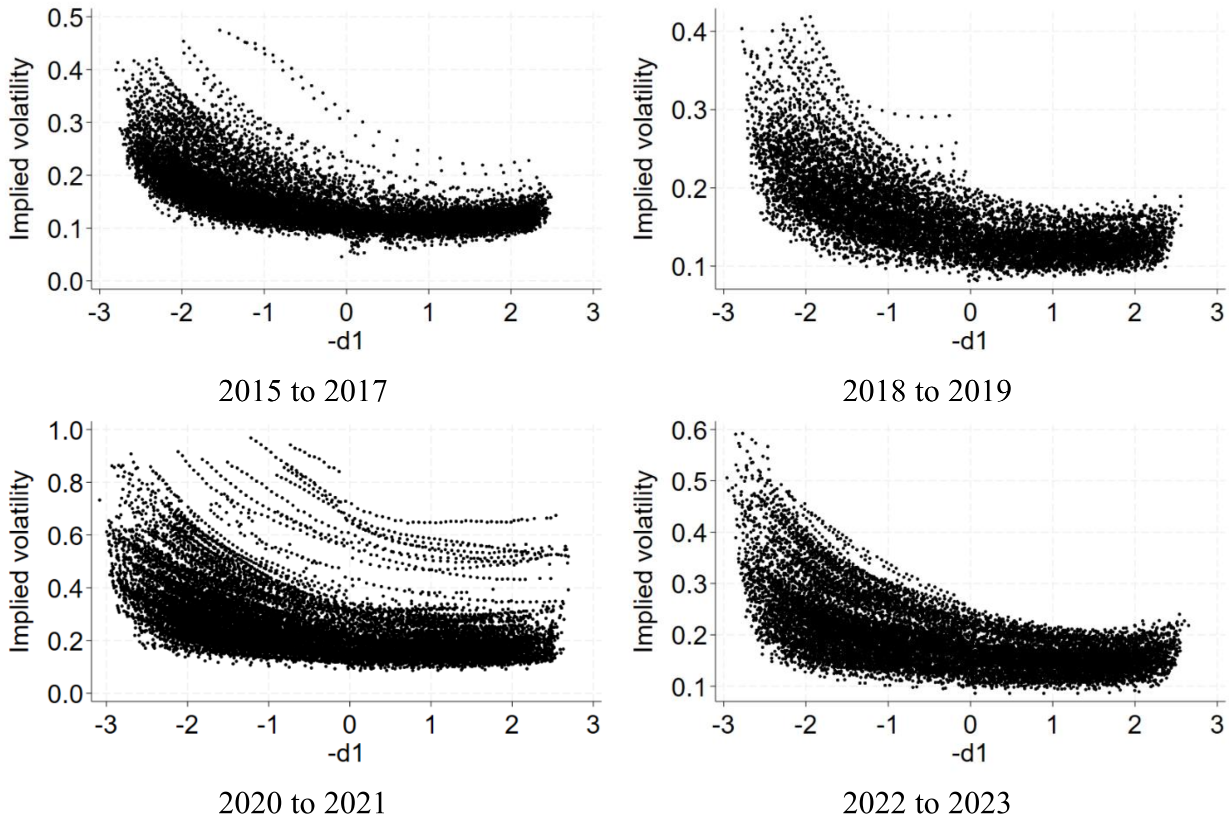
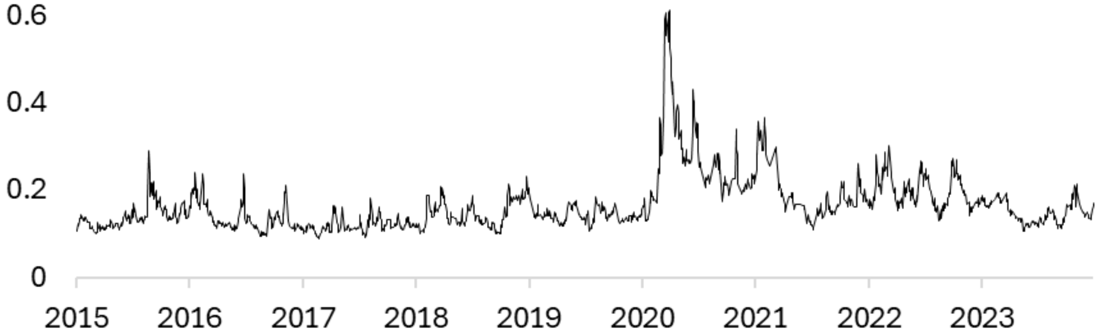
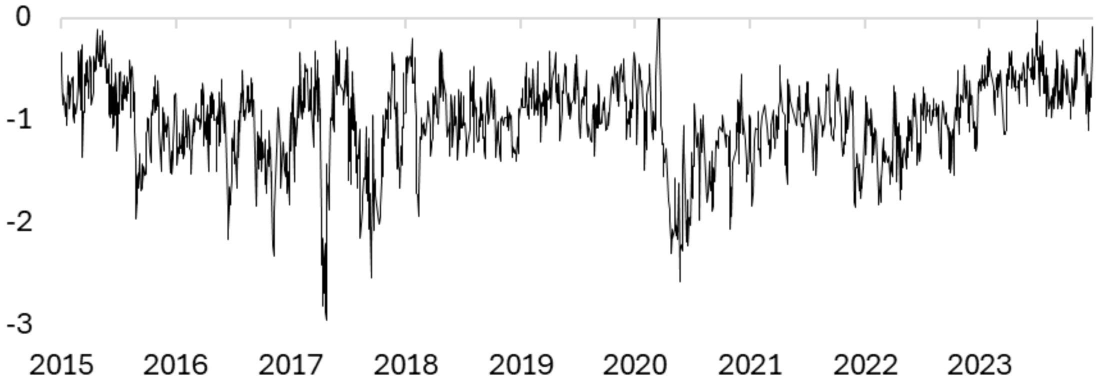


Figure 1. Black-Scholes implied volatility by moneyness and time period

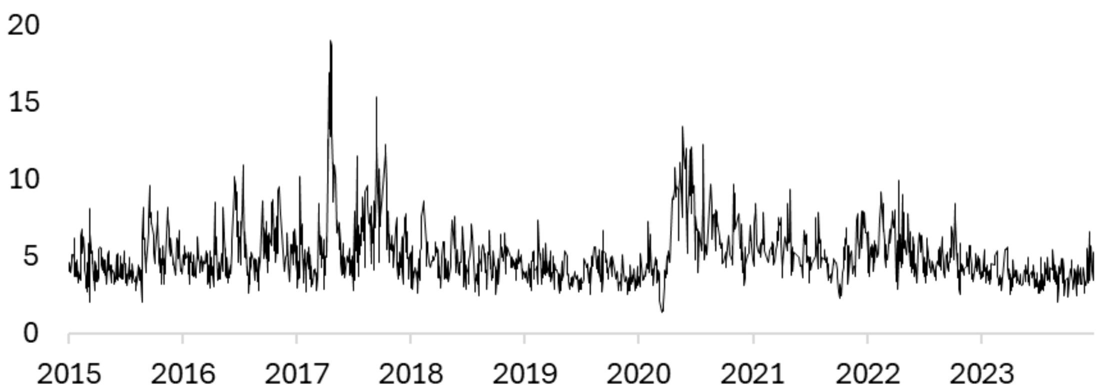
Notes: This figure illustrates the level of Black-Scholes implied volatility for out-of-the-money options, categorized by moneyness and time period. To enhance the clarity of the implied volatility curve, we focus exclusively on options with a time to maturity between 15 and 45 calendar days, consistent with the one-month time to maturity assumed in this study. Moneyness is measured using Black-Scholes d_1, which has been sign-switched to adhere to the convention of displaying puts on the left and calls on the right.



Panel A. Implied volatility estimate



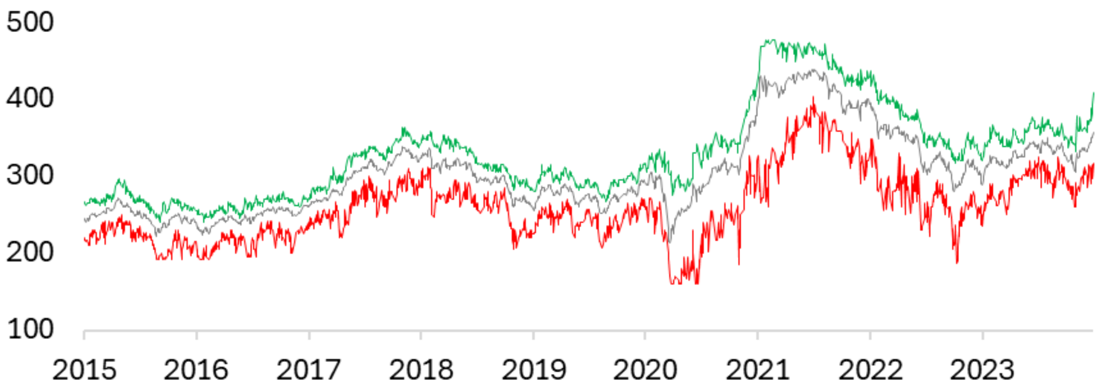
Panel B. Implied skewness estimate



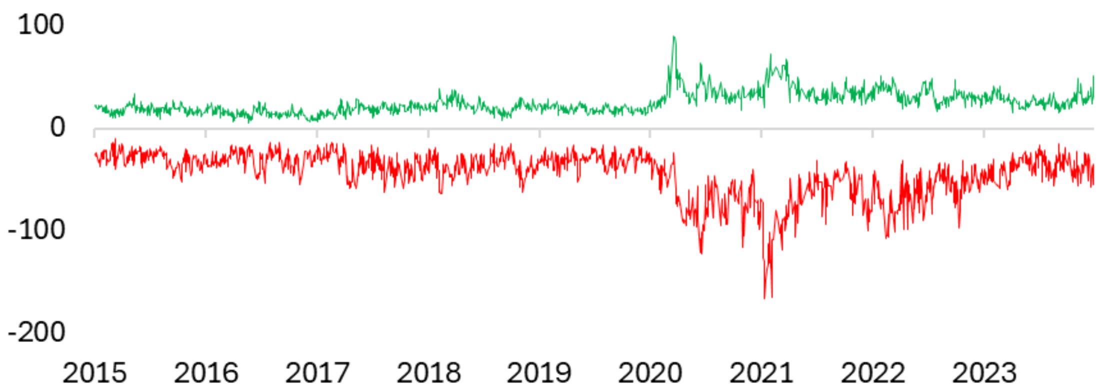
Panel C. Implied kurtosis estimate

Figure 2. Dynamics of implied moment estimates

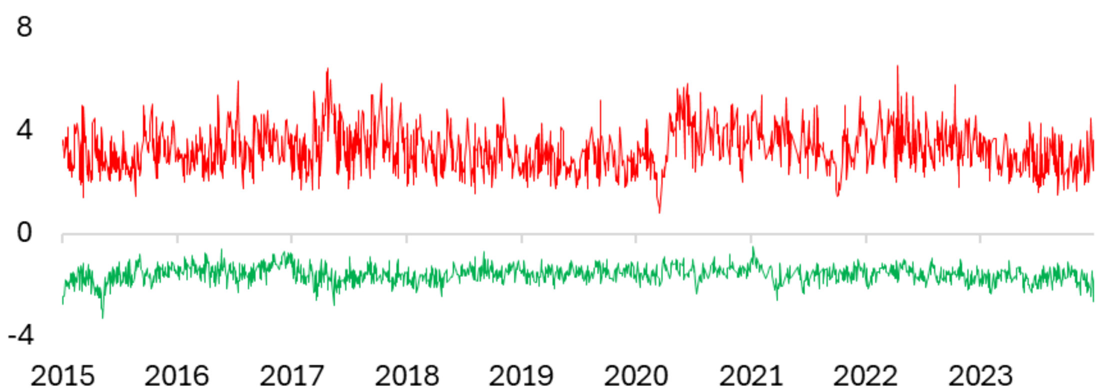
Notes: This figure depicts the daily dynamics of implied moment estimates, calculated using the BKM estimators, for a fixed one-month time to maturity across the full sample period from 2015 to 2023. We do not consider any truncation error correction method for implied moment estimation.



Panel A. Strike price level



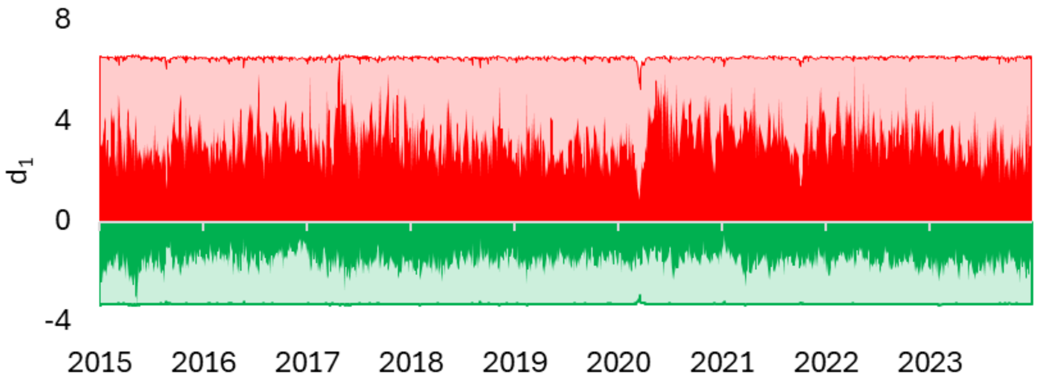
Panel B. Strike price gap from underlying price



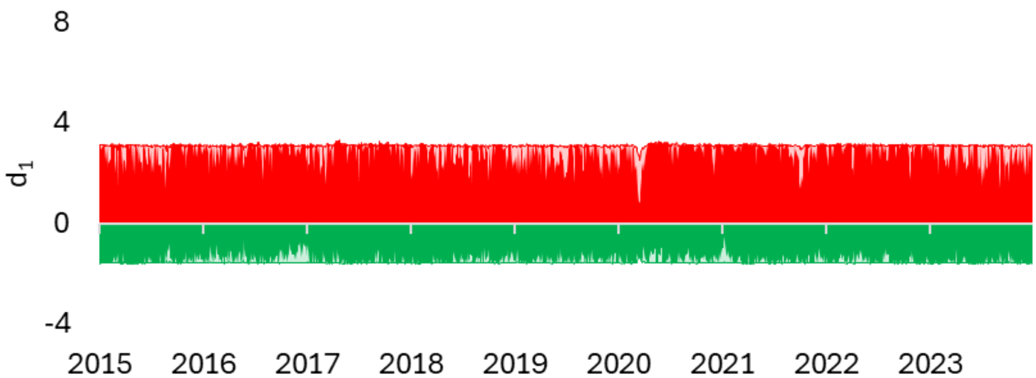
Panel C. Black-Scholes d_1

Figure 3. Dynamics of the integration domain

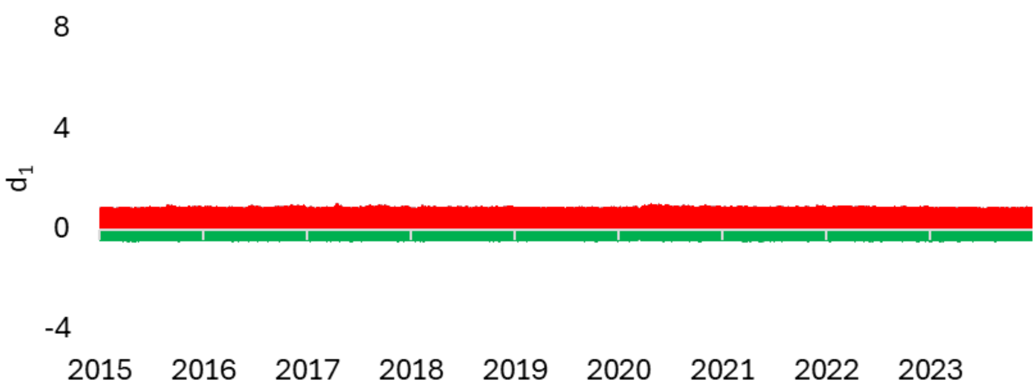
Notes: This figure illustrates the time-series dynamics of the integration domain after data filtering. The minimum and maximum endpoints of the integration domain are defined as $K_{\min}$ and $K_{\max}$ in Panel A, $K_{\min} - S$ and $K_{\max} - S$ in Panel B, and $d_1(K_{\min})$ and $d_1(K_{\max})$ in Panel C. Here, $K_{\min}$ and $K_{\max}$ represent the minimum and maximum strike prices of the integration domain, S is the underlying price, and d_1 refers to the Black-Scholes d_1.



Panel A. 0 percent stabilization



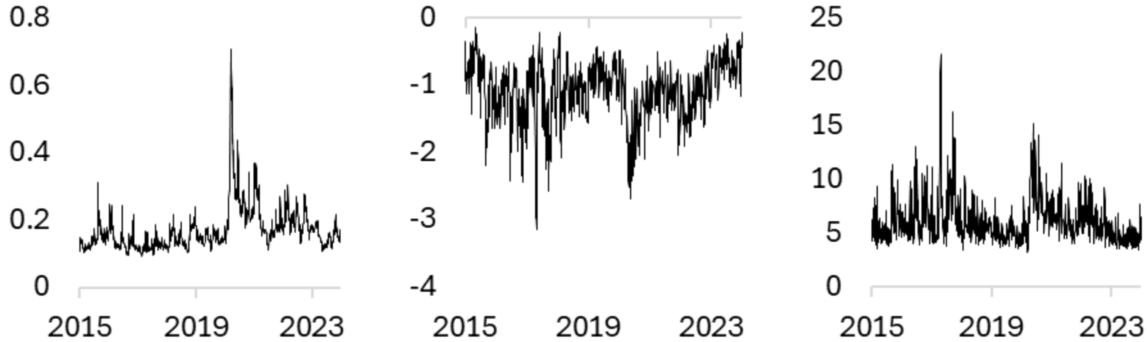
Panel B. 50 percent stabilization



Panel C. 100 percent stabilization

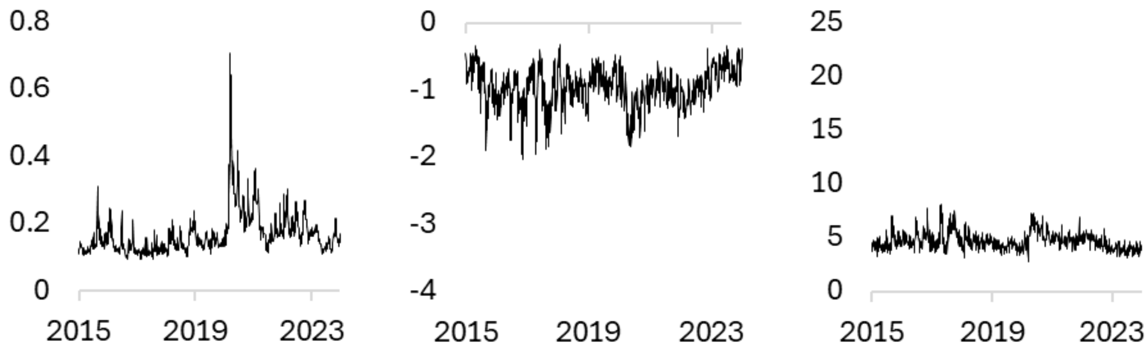
Figure 4. Integration domain after domain stabilization

Notes: This figure depicts the dynamics of the integration domain endpoints following the impliementation of domain stabilization at varying intensity levels. Under *n* percent stabilization, OTM option price observations are excluded if their d_1 values fall below the n^{th} percentile of the daily minimum d_1 values or exceed the $(100 - n)^{\text{th}}$ percentile of the daily maximum d_1 values. Subsequently, the flat extrapolation method proposed b Jiang and Tian (2005) is applied up to those percentiles. Dark-shaded areas represent portions of the integration domain with observed OTM option prices, while light-shaded areas indicate regions with extrapolated prices.



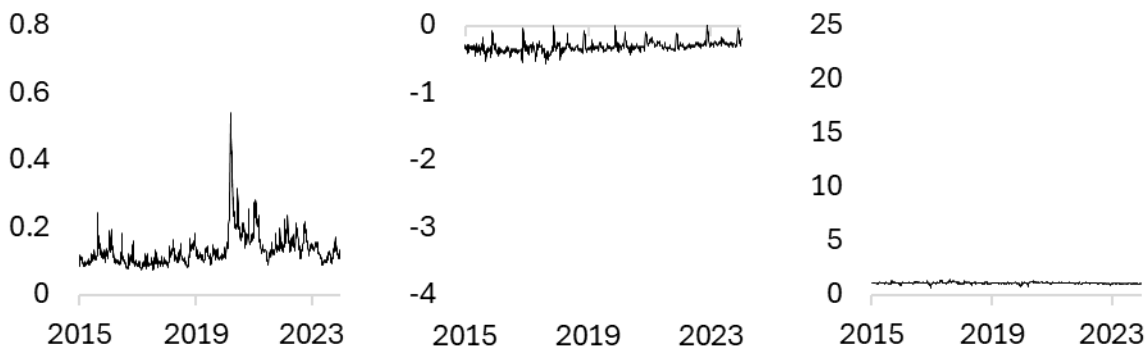
Implied volatility estimate | Implied skewness estimate | Implied kurtosis estimate

Panel A. 0 percent intensity



Implied volatility estimate | Implied skewness estimate | Implied kurtosis estimate

Panel B. 50 percent intensity



Implied volatility estimate | Implied skewness estimate | Implied kurtosis estimate

Panel C. 100 percent intensity

Figure 5. Dynamics of implied moment estimate after domain stabilization

Notes: This figure shows the dynamics of implied moment estimates after applying domain stabilization at different intensity levels.